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Newton's Fractals on Surfaces via Bicomplex Algebra

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GOALS AND MOTIVATIONS

Procedural generation of 3D and 4D **fractals**.

Fractal patterns are very useful for **volumetric rendering** and **surface decoration**.

STATE OF THE ART

Fractals are widely used for 2D visualization.

Common approaches for 3D and 4D are **fractal noise** functions (*e.g.* Perlin, Simplex).

Previous approaches based on bicomplex numbers have been used to generalize **escape-time fractals** (*e.g.* Mandelbrot, Julia).

BICOMPLEX NUMBERS

Bicomplex numbers form a **4D algebra** with operators and differentiation rules analogue to the complex numbers.

The **basis for the bicomplex space** is the following

$$\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{i} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$$

$$\mathbf{j} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad \mathbf{k} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

There is a relation between complex and bicomplex **polynomial roots**

$$\alpha_p = \exp\left(2\pi i \frac{p}{n}\right) \qquad \beta_{pq} = \frac{(\alpha_p + \alpha_q)\,\mathbf{1} + (\alpha_p - \alpha_q)\,\mathbf{k}}{2}$$

solves solves

$$c^n - 1 = 0 \qquad z^n - 1 = 0$$

REFERENCES

[1] H.O. Peitgen, D. Saupe, Y. Fisher, M.F. Barnsley, Association for Computing Machinery Special Interest Group on Graphics, M. McGuire, B.B. Mandelbrot, R.L. Devaney, and R.F. Voss. 1988. *The Science of Fractal Images*. Springer New York.

[2] M.E. Luna-Elizarrarás, M. Saphiro, D.C. Struppa, and A. Vajiac. 2012. *Bicomplex Numbers and their Elementary Functions*. Cubo (Temuco) 14 (00 2012), 61 – 80.

[3] X.-Y. Wang and W. Song. 2013. *The generalized M–J sets for bicomplex numbers*. Nonlinear Dynamics 72 (2013), 17–26.

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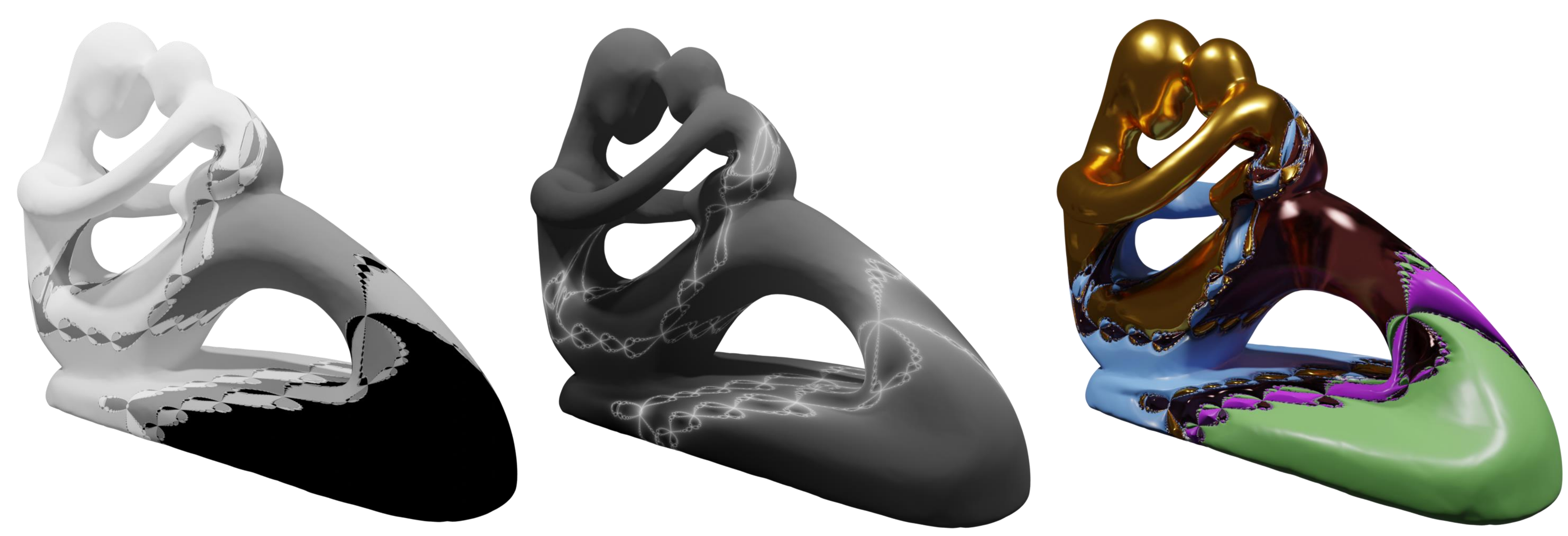
OUR APPROACH

Generalization of the **Newton's fractals** in the bicomplex setting.

Efficient **pixel shader** implementation of the needed algebraic operations.

Smooth convergence gradient for additional details in the pattern.



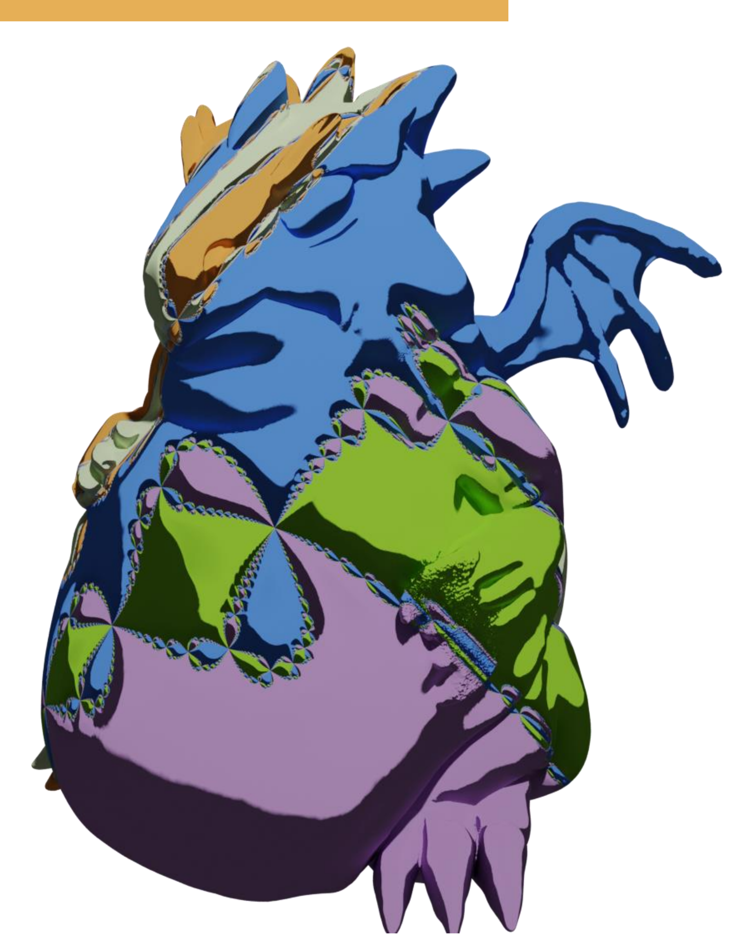
Efficient computation of the Newton-Raphson method.

$$z_{k+1} = z_k - \frac{f(z_k)}{\frac{\partial f(x_k)}{\partial z}} = z_k - \frac{z_k^n - 1}{n\, z_k^{n-1}} = \frac{1}{n}\left((n-1)z_k + z_k^{1-n}\right)$$

Smooth convergence gradient for additional detail in the pattern.

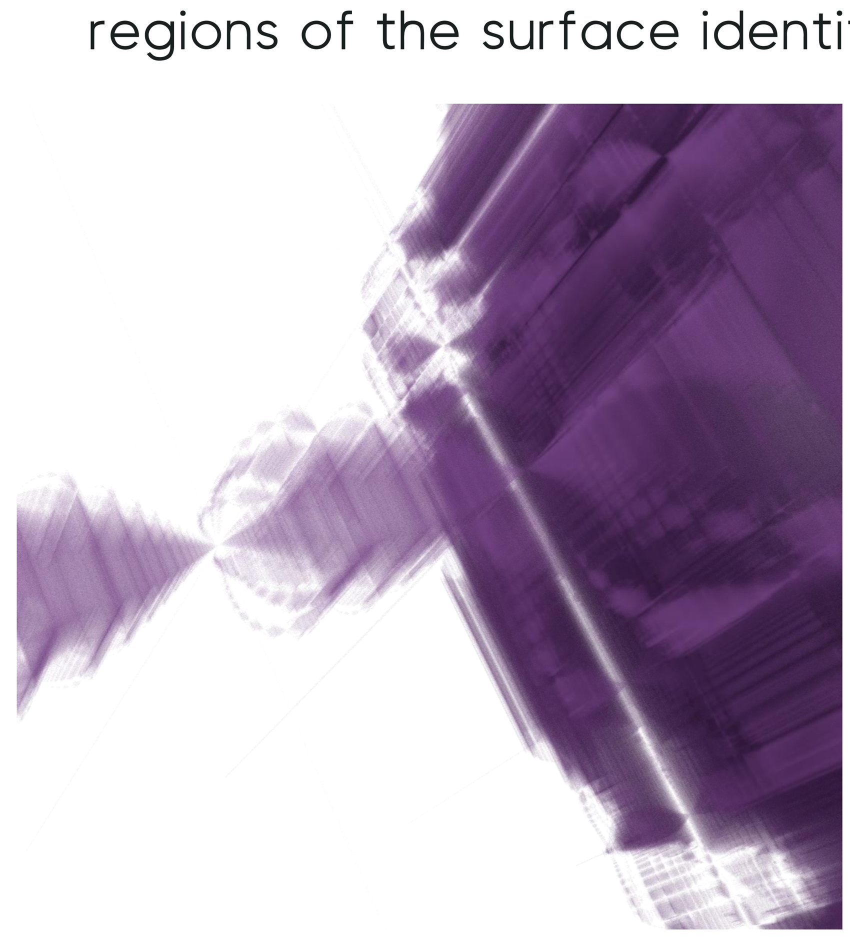
$$P - \sum_{k=0}^{P} \frac{1}{1 + \exp(\delta_k + \theta) - \exp(\theta)}$$

APPLICATIONS



Procedurally generated textures can be easily used as **decoration pattern** for surfaces. The convergence gradient gives good results when used as a **bump map**.

The regions identified by the solutions of the bicomplex polynomials can be used for **masking**. **Different materials** can be applied to different regions of the surface identified by the fractal.



Solution regions can also be visualized in 3D by means of **volumetric rendering**. These regions can be used for interesting 3D visualizations, as well as discrete **3D filters**.