Fine Water with Coarse Grids: Combining Surface Meshes and Adaptive Discontinuous Galerkin

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Simulating water for visual effects demands a high resolution surface with precise dynamics, but a fine discretization of the entire fluid volume is generally inefficient. Prior adaptive methods using octrees or unstructured meshes carry large overheads and implementation complexity. We instead show the potential of sticking with coarse regular Cartesian grids, using detailed cut cells at boundaries, and introducing a p-adaptive Discontinuous Galerkin (DG) method to discretize the pressure projection step in our fluid solver. This retains much of the data structure simplicity of regular grids, more efficiently captures smooth parts of the flow, and offers the flexibility to increase resolving power where needed.

We use the El Topo library [Brochu and Bridson 2009] for tracking the water surface with an explicit triangle mesh. Explicit surface tracking has excited much interest, offering unmatched efficiency in following thin and detailed features — but without a matching discretization of the flow physics or appropriate regularization, fine-scale features may behave badly or even cause instability. We also use FLIP particles for advection, including the surface mesh vertices. This novel combination greatly reduces numerical dissipation found in previous fluid solvers using explicit surface tracking.

Adaptive Discontinuous Galerkin

The Discontinuous Galerkin approach to discretizing partial differential equations is closely related to the famous finite element method. For DG, the domain is partitioned into cells, and some (typically polynomial) approximation space is assumed within each cell. DG allows each cell to have different approximation functions regardless of discontinuities across cell boundaries, and the cells need not be simple shapes. Of the many DG methods in the literature, we use the Local Discontinuous Galerkin (LDG) method [Cockburn et al. 2005], which is well-studied, flexible, and has no mesh-dependent parameters. DG has already been used in animation for elasticity [Kaufmann et al. 2009].

For the elements of the DG simulation, we use the cut cells produced by intersecting the Cartesian grid cells with the water volume defined by the surface mesh. This discretizes the PDE in the detailed shape of the high resolution surface - directly capturing thin sheets and other small features.

The principle technique of a p-adaptive approach is to use different approximation spaces in different cells. Within cells near the boundary, we use high-degree polynomials for high-fidelity solutions. Away from the surface, where less detail is needed, we use low-degree polynomials for efficiency. This p-adaptive approach allows us to use a coarse regular grid for the whole domain, while still achieving fine-scale details where desired.

In contrast to h-adaptive techniques, such as octrees, p-adaptation is an unexplored avenue within computer graphics. For smooth functions, p-refinement can produce more accurate results with fewer degrees of freedom than h-refinement. For non-smooth functions, h-refinement is more appropriate, but water simulations typically have smooth solutions. The major exception is when topology changes occur and introduce a nearly-discontinuous velocity. Even in this case, we find our p-adaptive approach works well.

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SIGGRAPH 2013, July 21 – 25, 2013, Anaheim, California.

ACM 978-1-4503-2261-4/13/07

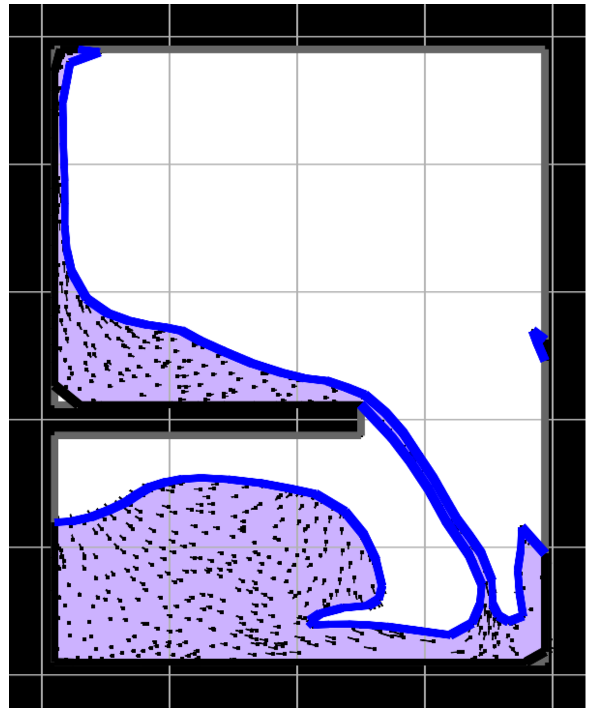

Figure 1: *In this water simulation we used a 4x5 grid with quartic polynomials for velocity in each cell. Advection is handled by FLIP particles - shown with black ticks. Notice the very thin sheets, well below the grid resolution.*

We apply several techniques to ensure good conditioning of the matrix produced by LDG. First, we merge cut cells that have very small volume with a larger adjacent cell. Because El Topo does not produce arbitrarily thin sheets, such a merger is always possible. Second, we choose a basis that is adapted to the cut cell shape by fitting a simplex to each cut cell and using the Lagrange basis with nodal points distributed on this simplex.

Our 2D implementation shown in Figure 1 captures thin sheets, splashes, and overturning waves all well below the grid resolution.

References

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