Mapping Chaos

David Trowbridge[*]   Micah Dowty[†]

1 Introduction

Iterated Function Systems are a celebrated class of dynamical systems in the computer graphics world. By defining a constrictive set of functions and recursing, beautiful fractal images can be created. Classically these systems use a set of affine transformations, such as in Sierpinski's Gasket. More complex versions of these can be seen in the popular "flame" fractals and single-orbit chaotic maps. When any of these systems is computed the result is a density field which can then be rendered in any number of ways.

The traditional imaging techniques used to visualize these types of systems are usually very simple, allowing only a few thousand points to be drawn. However, with high-dynamic-range image processing techniques these images can contain many millions of points, exposing far more internal structure.

2 Exposition

Generally the literature discussing chaotic maps and other systems which produce density fields is full of greyscale images created where each pixel is darkened as the point or points move around the image region. This technique is often sufficient for analyzing the structure of a map but does not provide a sound basis for the use of the map as an artistic entity. The structure of the attractor is only minimally visible using such techniques, because a long iteration run will quickly result in a completely black image.
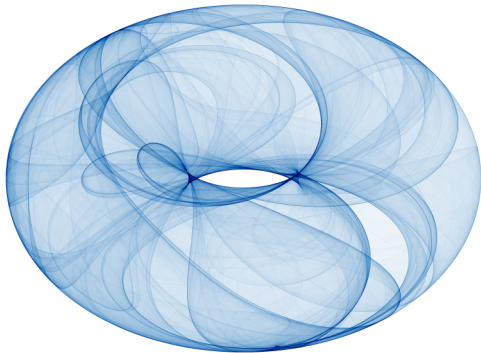

Figure 1: A complex attractor rendered as a two-dimensional histogram. Fine details are made easily visible by color and shading. This image contains approximately 100 million data points.

Instead, our technique treats the entire image region as a two dimensional histogram. Unlike previous techniques where each iteration

is plotted by darkening a pixel directly, this histogram technique maintains total image intensity as more iterations are performed. As more samples accumulate in the histogram the image simply gets more detailed. Instead of an image containing only a few thousand iterations of the map, millions or billions of points can be plotted with no loss of detail.

Like other rendering techniques that progressively refine an image, this histogram method performs particularly well in interactive applications. Our software for interactively browsing parameter space runs iterations for each frame only until a predetermined amount of time has elapsed, displaying a low quality but complete image nearly instantaneously. Instead of waiting for a preset number of iterations to complete, the user can zoom through parameter space quickly, viewing only what the CPU has time to render. As soon as the user stops changing parameters, the image quality will begin to improve.

Because a particular histogram bucket may contain tens of thousands of hits, high-dynamic-range image processing techniques can be applied. This includes gamma correction, exposure control, and color interpolation at the histogram's full precision. By clamping each color component as late as possible in the rendering process, the color can provide a visual hint at the higher dynamic range of the image. This is typically seen as a convergence towards either black or white depending on the colors chosen.

A histogram of infinitely small points can not be antialiased using standard coverage techniques, but aliasing artifacts can be reduced by oversampling– using multiple histogram buckets per pixel.

3 Conclusion

Treating histograms as high-dynamic-range images creates the opportunity to convey far more information in a single image than conventional techniques. Simple greyscale images containing only a few thousand points can only begin to hint at the true complexity of systems such as chaotic maps. This technique is applicable to any large data set that can be represented as a density field. It has shown to work particularly well in the visualization of chaotic attractors and bifurcation diagrams of chaotic systems.

The possibility of interactivity enables this technique to be used not only for data visualization but as an artists tool. In particular, the ability to interactively explore the parameter space of a chaotic map allows the discovery of beautiful attractors which can then serve as textures or other feed material. Possible future directions of this technique involve the use of algorithms to automatically detect and avoid parameter sets which result in non-chaotic attractors such as fixed points or computer vision techniques for identifying parameters likely to be visually pleasing. These new techniques make it easy for artists to exploit the beauty of chaotic systems and gives researchers a much more effective visualization tool.

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[*]e-mail: trowbrds@cs.colorado.edu

[†]e-mail: micah@navi.cx