# Ocean Waves Animation using Boundary Integral Equations and Explicit Mesh Tracking

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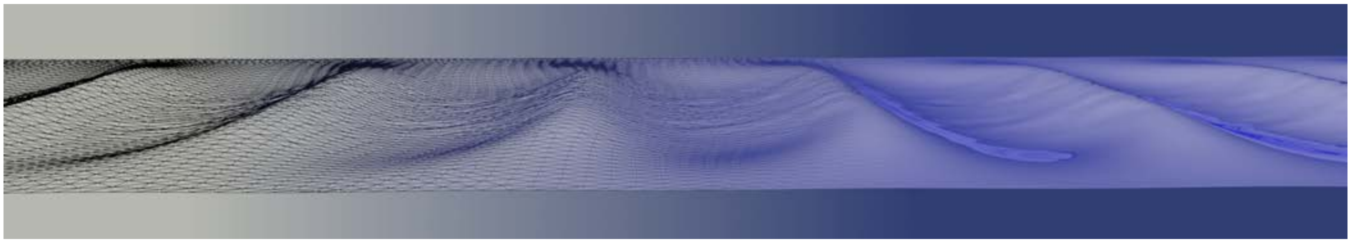

**Figure 1:** *Series of deep ocean waves*

## 1 Introduction

Ocean waves are a common animation challenge. To achieve natural-looking and rich detail, physical simulation of one kind or another has generally been adopted. For non-overturning waves without significant interaction with boats or other solids, or just to provide a good ripple-scale animated displacement texture, Tessendorf's extremely efficient FFT-based solver [2001] is typically used. However, its realism falters for stormy oceans and doesn't account for the presence of large solids in the water.

For full-fledged interaction with solids and for overturning waves with splashes, 3D simulation of the free-surface Navier-Stokes equations is the main alternative. However, this comes with a significant performance cost, as a large volume of water beneath the surface must be discretized and solved: though it is not directly rendered, the effect of the depth is plainly visible in terms of wave speeds and dispersion. The finite simulated domain generally has to be much smaller than the field of view in rendering as well, requiring nontrivial effort to convincingly blend the 3D region into some sort of continued surface geometry out to the edges of the view.

Our method brings the following novel contributions to the animation of water wave phenomena, bridging the gap between Tessendorf's waves and volumetric fluid simulation:

- a surface-only memory-efficient method for computing deep ocean waves,
- a boundary integral formulation better fitted to coarse explicit meshes, including solid boundaries,
- a method which implicitly satisfies the infinite depth of the deep ocean without needing a large simulation domain,
- optimal scaling of computation time using advanced hardware and algorithms.

The core of our novel animation technique is derived from methods detailed in engineering literature that are found in papers such as Xue et al. [2001] and Grilli et al. [2001] where numerical methods solving wave dynamics rely on boundary integral equations (BIEs) to do surface-only computations to propagate wave dynamics. These techniques allow solid body interaction with submerged and partially-submerged objects. We employ explicit mesh tracking using the El Topo library [Brochu and Bridson 2009] to represent the mesh in a robust and adaptive manner. We also employ advanced acceleration techniques by using GPU hardware and advanced algorithms such as the Fast Multipole Method (FMM).

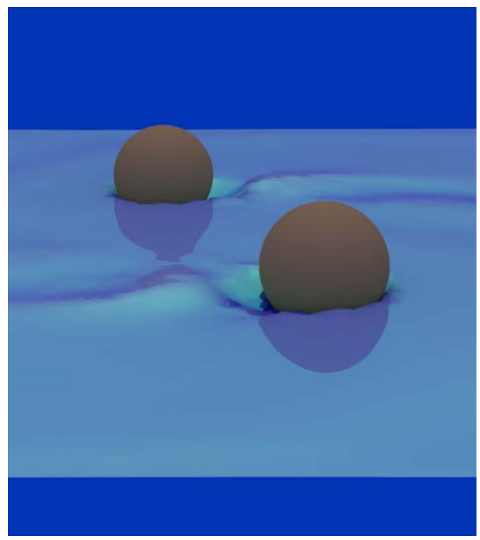

**Figure 2:** *Wake Behind Two Spheres*


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## References

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