Camera Calibration by Recovering Projected Centers of Circle Pairs

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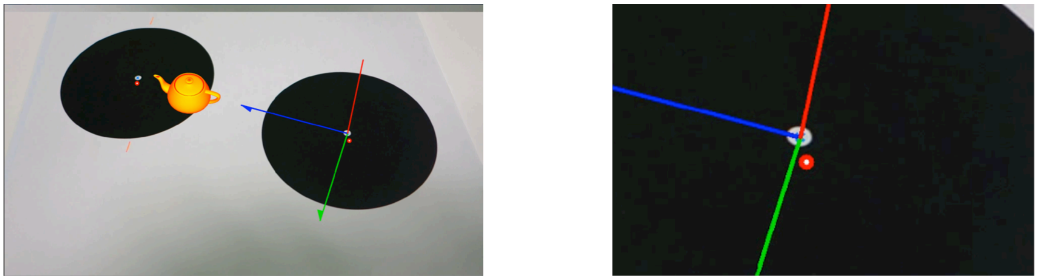

(a) (b)

Figure 1: *(a) Experimental image shows result of calibration with our algorithm using co-planar circle-pairs. (b) Zoomed up center part of the circle on the right in (a) shows that the projected center (where white circle is ground truth) was estimated accurately and applied in camera calibration. For comparison, the center of ellipse whose parameters are calculated by FitEllipse function in OpenCV is also showed here as a red dot.*

Abstract

In this paper, we present a convenient method for camera calibration with arbitrary co-planar circle-pairs from one image. This method is based on the accurate recovery of the projected centers of the circle pairs using a closed-form algorithm.

Keywords: computer vision, camera calibration

Concepts: **• Computing methodologies ~ Image and video acquisition;** *Camera calibration;*

1 Introduction

Camera calibration is a fundamental requirement for computer vision based applications. Projective properties of circular features including projected centers of circles have been deeply researched and realized as solutions for camera calibration. Kim proposed a method of using concentric circles to recover affine and Euclidean structures with the help of the projected centers of concentric circles [Kim et al. 2002]. Zhao estimated the projected centers of planar circle grids and used them for camera calibration [Zhao et al. 2010]. These methods simplify the calibration procedures compared with plane based or point correspondence based calibration approaches by operating a simple calibration object, and they show comparable performance.
However, restrictive constraints still exist that hinder them from being applied to broader practical use, since Kim needed two views of concentric circles with known size and Zhao needed at least three circles. Some approaches have been proposed [Chen et al. 2004, Chen et al. 2014] to overcome the remaining constraints with one view of two circles to solve the calibration problem. However, this is achieved at the cost of losing accuracy. Since, with less information, projected center cannot be recovered, these approaches use the centers of ellipses in images as the projected centers of circles. This is not correct in general and is only an acceptable approximation when the ellipses in images are small enough.
In this paper, we propose a convenient and accurate camera calibration method. The primary contributions are:

1. Camera parameter estimation with arbitrary co-planar circle-pairs from one single image with the help of projected centers.
2. Algorithm for recovering the projected centers of co-planar circle-pairs.

2 Recovering projected centers

The existing methods of Kim and Zhao need concentric circles or three circles to analyze projected centers. They use abstract or invisible properties of the quadric form of circles, such as the sign of an eigenvalue, the direction of an eigenvector, the rank of a matrix.
With the common tangents of circle pairs as the key helper, we developed an algorithm for recovering the projected centers of two co-planar circles. Our algorithm uses real, visible, and concrete geometrical features such as points and lines for describing itself and for representing the intermediate results; therefore, the calculation process can be easily visualized (**Figure 2**).

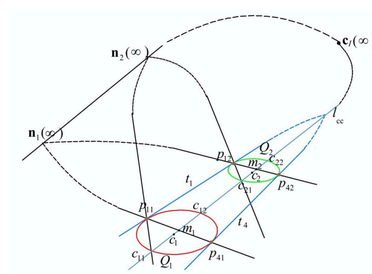

Figure 2: *Illustration that shows every step of our algorithm.*

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2.1 Common tangents of co-planar circle pairs

Consider a camera coordinate system $\mathbb{C}$ that of which the origin is the optical center and the z-axis is the optical axis. Let Q_1 and Q_2 be the quadric form of circle- pairs that appear as ellipses on the image plane in $\mathbb{C}$. Then, the envelops of the two circles are Q_1^{-1} and Q_2^{-1}. The common tangents that are the tangent lines of the both circles can simultaneously can be determined by finding the common "points" of Q_1^{-1} and Q_2^{-1}. Since two circles do not enclose each other, there are always two common tangents that of which the both circles are on the same side. We can call the common tangents of this kind as "outer tangents" which touch the circles at points p_{11}, p_{12}, p_{41}, and p_{42}.

2.2 Determining line passing through centers of circle pairs

Since the mid-point (m_1) of the line segment $p_{11}p_{41}$ and the mid-point (m_2) of the line segment $p_{12}p_{42}$ are on the line that passes through the two centers, we use m_1 and m_2 to determine that line. $p_{11}p_{41}$ and $p_{12}p_{42}$ are parallel in 3D space, so the vanishing point $n_{1(\infty)}$, which is the projected point at infinity, can be calculated as $(p_{11}\times p_{41})\times(p_{12}\times p_{42})$. Then, the distance $p_{41}m_1$ and $p_{42}m_2$ can be calculated according to the invariance of the cross ratio of four co-linear points as

$$p_{41}m_1 = p_{11}p_{41} \cdot p_{41}n_{1(\infty)} / (2p_{41}n_{1(\infty)} - p_{11}p_{41})$$

The mid-point m_1 is then calculated with

$$m_1 = p_{41} + p_{41}m_1/p_{11}p_{41}\times(p_{11} - p_{41})$$

$p_{42}m_2$ and m_2 can be calculated in the same way; then, the line that passes the two centers is obtained with $l_{cc} = m_1\times m_2$.

2.3 Determining centers of circle pairs

Once the line l_{cc} that passes through the two centers has been determined, we find the common points (c_{11}, c_{12}) of Q_1 and l_{cc} and the common points (c_{21}, c_{22}) of Q_2 and l_{cc}. Since lines $c_{11}p_{11}$ and $c_{21}p_{12}$ are parallel, we can calculate another projected point at infinity $n_{2(\infty)}$ with them as $(c_{11}\times p_{11})\times(c_{21}\times p_{12})$.
Then the line at infinity $l_{(\infty)} = n_{1(\infty)} \times n_{2(\infty)}$.
Since the projected centers of the two circles (c_1, c_2) and the line at infinity are in a pole-polar relationship, the two centers can be calculated with $l_{(\infty)}$ as follows.

$$c_1 = Q_1^{-1} l_{(\infty)}$$

c_2 can be calculated in the same way.

3 Camera parameters estimation

Define a world coordinate system $\mathbb{W}$ of which the z-axis is the normal vector $\mathbf{n}$ of the plane where two circles reside and the x-axis is the vector $\mathbf{x}$ parallel to l_{cc}. The rotation matrix $\boldsymbol{\mathcal{R}}$ between $\mathbb{W}$ and $\mathbb{C}$ can be described as $[\mathbf{x} \quad \mathbf{n}\times\mathbf{x} \quad \mathbf{n}]$. The origin $\mathbf{o}$ of $\mathbb{W}$ is at one of the two centers.

3.1 Estimating rotation matrix and focus length

To determine $\boldsymbol{\mathcal{R}}$, a supporting coordinates system $\mathbb{S}$ is defined with the same rotation as $\mathbb{W}$; thus, the rotation matrix between $\mathbb{S}$ and $\mathbb{C}$ is also $\boldsymbol{\mathcal{R}}$. The origin of $\mathbb{S}$ is the same as $\mathbb{C}$ at the optical center. First, consider one of the two circles, which can be described by using quadric form as Q_{s1} in $\mathbb{S}$ and projecting to the image plane with perspective projection as Q_1. Since $\mathbb{S}$ and $\mathbb{C}$ have a common origin, the transform between Q_{s1} and Q_1 only involves $\boldsymbol{\mathcal{R}}$ as

$$kQ_1 = \boldsymbol{\mathcal{R}} Q_{s1} \boldsymbol{\mathcal{R}}^T$$

, where $k \neq 0$ is a scale factor. This equation cannot be solved for all the variables in $\boldsymbol{\mathcal{R}}$. However, since the normal vector $\mathbf{n}$ is the third column vector of $\boldsymbol{\mathcal{R}}$, by converting Q_1 to a diagonal matrix and substituting it into the equation, two possible values of $\mathbf{n}$ can be obtained. Applying the same algorithm on the other circle Q_{s2} and Q_2, another two possible values of $\mathbf{n}$ can be calculated. The normal vector calculated from two circles should be in the same direction since the two circles are co-planar; thus, $\mathbf{n}$ can be uniquely selected from the possible values by finding the smallest cross product of each pair, and the focus length can be simultaneously estimated during the process [Chen et al. 2004].

3.2 Estimating origin of world coordinate

To estimate the location of each center of the two circles in $\mathbb{C}$, the 3D position of the centers in $\mathbb{S}$ is required. It can be calculated with the homogeneous coordinates and z-axis distance z of the centers in $\mathbb{S}$ [Chen et al. 2014]. The homogeneous coordinates $(x_{s1}, y_{s1}, 1)$ and $(x_{s2}, y_{s2}, 1)$ in $\mathbb{S}$ can be calculated with projected centers $c_1 = (x_1, y_1)$ and $c_2 = (x_2, y_2)$ estimated in **Section 2.3** as $\boldsymbol{\mathcal{R}}^T[x_1 \quad y_1 \quad 1]^T$ and $\boldsymbol{\mathcal{R}}^T[x_2 \quad y_2 \quad 1]^T$, respectively. Then, we can have

$$z = d/\sqrt{(x_{s1}-x_{s2})^2 + (y_{s1}-y_{s2})^2}$$

, where d is an arbitrary scale that represents the distance between the centers in $\mathbb{S}$. Then, 3D positions $\boldsymbol{c_{s1}}$ and $\boldsymbol{c_{s2}}$ of the centers in $\mathbb{S}$ can be calculated as $z\,[x_{s1} \quad y_{s1} \quad 1]^T$ and $z\,[x_{s2} \quad y_{s2} \quad 1]^T$. The origin $\mathbf{o}$ of $\mathbb{W}$, which is one of the centers, is obtained by transforming $\boldsymbol{c_{s1}}$ or $\boldsymbol{c_{s2}}$ to $\mathbb{C}$ as $\boldsymbol{\mathcal{R}c_{s1}}$ or $\boldsymbol{\mathcal{R}c_{s2}}$. Finally, the matrix that transforms points in $\mathbb{W}$ to $\mathbb{C}$ is given as follows:

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{\mathcal{R}} & \mathbf{o} \\ 0_3^T & 1 \end{bmatrix}$$

4 Conclusion

We proposed a camera calibration algorithm using circle pairs with their projected centers. For estimating the accurate position of projected centers, we developed a straightforward and non-trial method using visible and concrete geometrical features.

References

KIM, J. S., KIM, H. W., AND KWEON, I. S. 2002. A Camera Calibration Method using Concentric Circles for Vision Applications. In *ACCV*, 23–25.

ZHAO, Z. AND LIU, Y. 2010. Applications of Projected Circle Centers in Camera Calibration. In *Machine Vision and Applications*, 853 21(3):301–307.

D. CHEN, R. SAKAMOTO, Q. CHEN, AND H. WU. 2014. Extrinsic camera parameters estimation from arbitrary co-planar circles. In *SIGGRAPH Asia 2014 Posters*, ACM, 28.

CHEN, Q., WU, H., AND WADA, T. 2004. Camera Calibration with Two Arbitrary Coplanar Circles. In *ECCV*, 521–532.