Burning The Medial Axis

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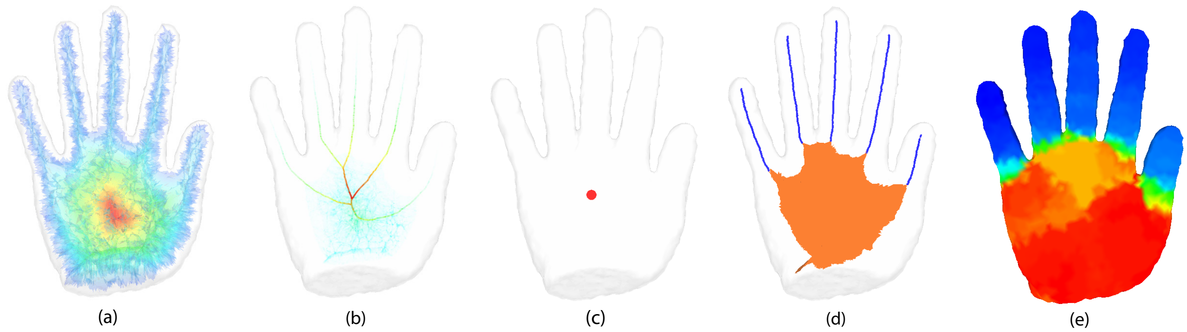

Figure 1: *Medial forms and applications on a hand model: (a) the medial axis colored by burn time (parts burned earlier have colder color and more transparency), (b) the medial curve colored by burn time, (c) the medial point, (d) a hybrid skeleton, (e) the shape-width function on the model surface.*

1 Background

Medial axis is a classical shape descriptor that is widely used in computer graphics, computer vision, and pattern recognition. Defined elegantly as the locus of points with multiple nearest neighbors on the object boundary, the medial axis preserves both the structure and topology of the object in a compact form - a geometry that has one lower dimension than the object itself.

In many applications, medial geometry at even lower dimensions are desirable. For example, the medial curve of a 3D object is useful for deformable shape matching and character animation. The medial point of an object is useful for object alignment and tracking. Although numerous heuristic approaches have been developed for computing medial curves and points of a 3D object, there has been little progress in developing a sound mathematical definition of these lower-dimensional medial geometry. To the best of our knowledge, the only definition of the medial curve of a 3D object was proposed in [Dey and Sun 2006]. However, their definition is quite different from that of the medial axis, and the defined medial curve is not guaranteed to preserve the topology of the object, which is a key property of the medial axis.

2 Method

We introduce a unified framework for defining and computing medial forms at various dimensions of a 3D object. The work builds on and generalizes a previous work for finding the medial point of a 2D object [Liu et al. 2011]. Intuitively, the medial form at a lower-dimension, M_d, is obtained by *burning* the medial form at a higher dimension, M_{d+1}. The fire starts on the boundary of M_{d+1} and propagates at uniform speed geodesically along M_{d+1}. Different fire fronts quench when they meet, and the quench sites form M_d. Burning the 3D object yields the medial axis (this way of defining the medial axis was first introduced by Blum [1967] as the *grass-fire analogy*). Burning the medial axis from its boundary yields the medial curve, the burning of which yields the medial point.

Our primary contribution is developing precise definitions of the burning process for a 3D object and demonstrating its theoretical properties. In particular, for an object represented as a union of balls, we show that the medial form M_d is generally d-dimensional and that each medial form is homotopy equivalent to the object. Our second contribution is developing discrete algorithms that compute approximate medial curves and points given an initial approximation of the medial axes as a triangular mesh (see Figure 1 (a,b,c)).

The medial forms, equipped with the burn time function, can give rise to new shape descriptors. The burn times on different medial forms M_d capture different shape characteristics, such as thickness ($d = 3$), width ($d = 2$), and length ($d = 1$). Also, while the medial forms are often noisy (as it is well know for the medial axis), we observed that the burn times are rather stable under boundary perturbations. We are currently exploring several derivative shape descriptors, including a *hybrid skeleton* made up of portions of the medial axis and medial curves that respectively capture plate-like and tubular object parts (Figure 1 (d)), and a metric on the object boundary that captures the width of the local shape (Figure 1 (e)).

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