Aspect-Ratio Based Triangular Mesh Smoothing

Xuan Huang
Bryn Mawr College
101 N. Merion Ave.
Bryn Mawr, PA 19010
xhuang1@brynmawr.edu

Dianna Xu
Bryn Mawr College
101 N. Merion Ave.
Bryn Mawr, PA 19010
dxu@cs.brynmawr.edu

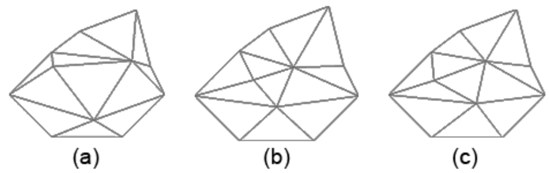

Figure 1: (a) Original (b) Angle-based (c) Aspect Ratio-based

CCS CONCEPTS

•Computing methodologies →Mesh models; Mesh geometry models;

KEYWORDS

Mesh smoothing, meshing quality improvement, aspect-ratio

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1 INTRODUCTION

Mesh quality improvement is an important problem with a wide range of practical applications. The element quality of a mesh heavily affects the results of numerical simulation done using that mesh. In the context of finite element mesh smoothing, vertex repositioning is the primary technique employed, where we allow tangential vertex motion only and the connectivity of the mesh is unchanged. Element quality is measured either by max/min angles or aspect ratio (longest edge over shortest), or both. We investigate a smoothing method focusing on improving aspect ratio. For triangle meshes this is in theory not significantly different from angle-based smoothing methods which have been widely studied. However many focus on improving minimum angles only and we believe that aspect ratio will lead to a more balanced improvement on both the minimum and maximum angles. In addition, we are also motivated by aspect ratio improvments for quadrilateral meshes, which are unrelated to angles.

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2 RELATED WORK

Existing methods usually divide into mesh modification via vertex insertion/deletion, edge/face swapping and remeshing [Dey and Ray 2010], or vertex repositioning without changing mesh connectivity [Amenta et al. 1997; Field 1988; Zhou and Shimada 2000]. Among those that do not modify mesh topology, Laplacian smoothing is the most commonly used because of its simplicity. In its most basic form, it moves each vertex to the barycenter of its neighboring vertices. It is a local method with very low computational cost, compared to the alternatives which are optimization-based [Chen and Holst 2011; Freitag 1997; Parthasarathy and Kodiyalam 1991]. The most closely related work is Zhou and Shimada's angle-based Laplacian smoothing [Zhou and Shimada 2000], which is a variant of Laplacian smoothing.

3 ASPECT-RATIO BASED SMOOTHING

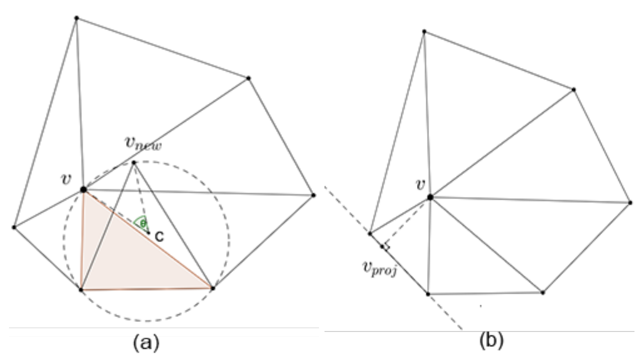

Figure 2: (a) Move v to v_{new}; (b) $\overline{vv_{proj}}$ is the shortest distance to star boundary

This section introduce a new smoothing method to improve aspect ratio. The idea is that inside the star region of each vertex, move the center vertex along the circumcircle of each incident triangle in a direction that will decrease the difference between each pair of adjacent interior edges. Each move must improve the aspect ratio of a triangle, because one of the two edges must be the shortest or longest edge of the face considered. Some restrictions are applied so that triangles already with good quality are not further modified and potentially sacrificed for worse ones.

The algorithm performs the following steps:

(1) As shown in Figure 2, for each vertex v not on the boundary there are k faces inside its star polygon. For each triangle if the two interior edges has a ratio higher than 1.5, calculate its circumcircle centered on C.
(2) Move v cw or ccw towards the longer incident edge.
(3) Let $\overline{vv_{proj}}$ be v's vertical distance to each star polygon edge. If the new position ends up decreasing the shortest distance from the vertex to the star polygon boundary, discard movement.
(4) Take the mean of all positions after processing all k faces and assign the v there. Iterate until the resulting positions converges.

4 RESULTS AND CONTRIBUTIONS

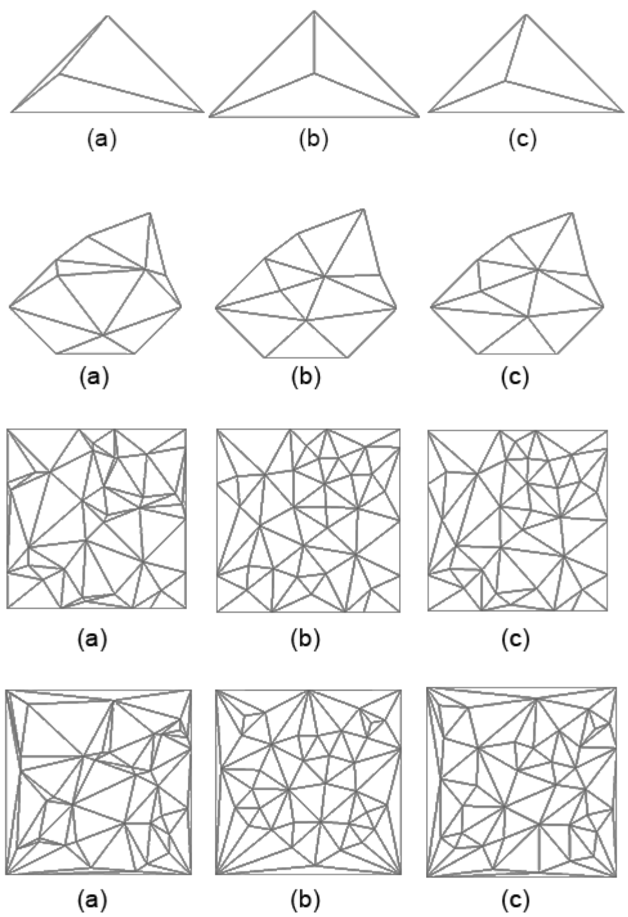

Figure 3: (a) Original (b) Angle-based (c) AR-based

As shown in the examples above, the max aspect ratio is improved over angle-based method in all cases. The algorithm might also provide some additional improvements on angles. It tends to recover seriously distorted areas and is also much less likely to result in invalid shapes compared to the angled-based method.

Table 1: Statistical comparison - best result underlined

	Original	Angle-based	Aspect Ratio-based
Simple Triangle	max: 168.465 min: 5.19443 max AR: 3.12348	max: 135 min: 22.5 max AR: 2.41421	max: 142.597 min: 14.9712 max AR: 2.35129
12 faces	max: 133.668 min: 10.0946 max AR: 5.70305	max: 115.484 min: 22.6975 max AR: 2.33943	max: 133.796 min: 22.969 max AR: 2.14802
40 faces (1)	max: 162.387 min: 2.48955 max AR: 11.1803	max: 132.203 min: 18.5365 max AR: 2.82876	max: 159.777 min: 8.16759 max AR: 2.56345
40 faces (2)	max: 174.549 min: 1.63658 max AR: 13.0384	max: 179.502 min: 0.248059 max AR: 3.42261	max: 174.147 min: 2.38678 max AR: 2.66102
1000 faces	max: 168.663 min: 1.03697 max AR: 48.8833	max: 138.854 min: 1.23549 max AR: 35.5843	max: 160.885 min: 2.20761 max AR: 19.4623

5 CONCLUSION AND FUTURE WORK

The aspect-ratio based method presented here generally improves the quality of any given triangular mesh, with a focus on improving worst-case aspect ratio. We are investigating additional ways of finding optional placements for the center vertex, including using a weighted average instead of the mean. In addition, we would like to see how the algorithm performs on improving aspect ratios for quadrilateral meshes, where the aspect ratios are not necessarily related to min/max angles.

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