Rendering Curved Spacetime in Everyday Scenes

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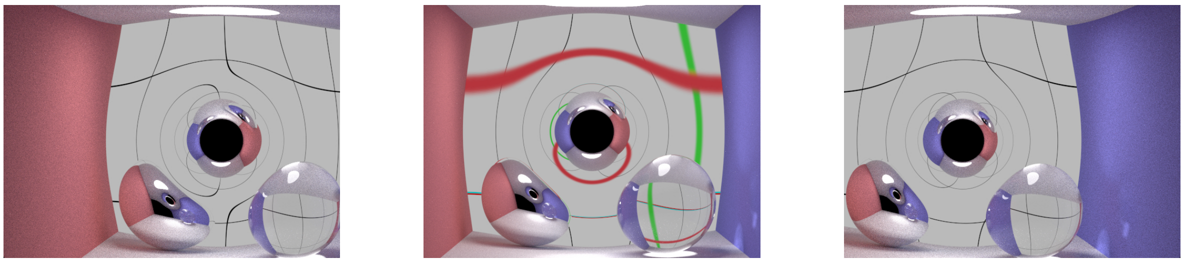

Figure 1: A room warps in the curved spacetime of a Schwarzschild black hole that sits in the center of the room. The warped scene is rendered by tracing light rays along the null geodesics of the curved spacetime as computed from the Einstein field equations of general relativity. The three images show how the warped room looks from the point of view of a moving observer. The grid on the back wall is deforming due to the distortion in spacetime. In the middle image, we show a version with few colored lines on the grid. This lets us track where the exact primary and secondary images are formed due to gravitational lensing. The red band on top of the black hole is the primary image of the red line, whereas the curved red line at the bottom that starts and ends at the black hole, is the secondary image of the red line. Similarly, the green line to the right is the primary image, and to the left is the secondary image.

ABSTRACT

We present a generic and principled Monte Carlo raytracing approach to visualizing curved spacetime. In contrast to earlier work, our method can trace rays in curved spacetime while resolving usual ray-object intersections. This not only allows us to visualize complex cosmological phenomena, but also create plausible visualizations of what happens when a black hole or a wormhole appears in a more known environment, like a room with regular specular and diffuse surfaces.

CCS CONCEPTS

• **Computing methodologies → Ray tracing**;

KEYWORDS

ray tracing, curved spacetime, general relativity, rendering

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1 MOTIVATION

The curving of spacetime, as predicted by the theory of general relativity, is present in the vicinity of any object that has mass. However, the concept of curved spacetime that is central to this theory stays elusive to a casual reader. We want to render a visualization of curved spacetime for cosmological scenes and in scenes at earth-like or everyday scales with regular objects that have diffuse and specular surfaces. We present, in this poster, a Monte Carlo path tracing method that allows us to visualize any arbitrary scene in any type of spacetime. We find the images that our renderer produces to be extremely useful in understanding the concept of curved spacetime and visualizing how the universe behaves in the presence of strong gravity. Earlier work in the area did not consider explicit ray-object interactions [James et al. 2015a,b] and considered very limited illumination models without any global illumination [Müller 2014; Weiskopf et al. 2004].

2 LIGHT PATHS IN CURVED SPACETIME

In order to generate the image of a curved spacetime, we must trace the geodesics along which light travels in such spacetimes. The geodesic written using index notation and Einstein summation convention, as is common in general relativity, is

$$\frac{d^2x^\lambda}{d\zeta^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{d\zeta}\frac{dx^\nu}{d\zeta} = 0 \tag{1}$$

Here μ, ν, λ represent the spacetime coordinates and can take the value $[0, 1, 2, 3]$. Repeated indices are summed from zero to three, as per the Einstein notation. $\Gamma^\lambda_{\mu\nu}$ are the Christoffel symbols. The first term represents the acceleration of the particle moving on the geodesic, if ζ is considered to be proper time. Here ζ represents the affine parameter that varies along the geodesic. We solve a

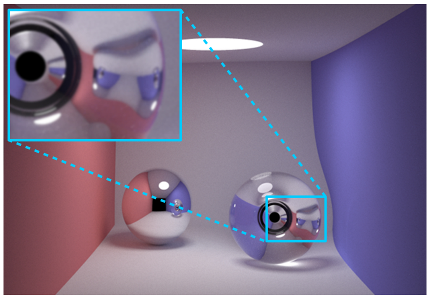

Figure 2: A black hole inside a refractive glass sphere curves the spacetime in a room.

Hamiltonian formulation of the above (Euler-Lagrange) geodesic equation, given a metric for the spacetime. In particular we have implemented solutions for the Schwarzschild (black hole) metric and the Ellis wormhole metric.

3 IMPLEMENTATION

In curved spacetime, we have to integrate the ray in time, and even for the simplest of objects, it is often not possible to find an analytic solution for the ray-object intersection. Hence, we propose and implement Algorithm 1 to solve this problem. Depending on scene complexity, the maximum number of iterations (MAXITER) for our experiments varies from 100 to 4000 and our starting time step size, dt, varies from 10.0 to 0.5. We use a RK4 integrator for our path tracer. We also sample our rays on a jittered grid to get an anti-aliased image. In general we found that a OpenCL implementation on nVidia Quadro M4000 GPU ran twice as fast a Intel fifth generation Xeon with 14 cores.

Algorithm 1 Path tracing & ray-object intersection in curved space

while number of iterations < MAXITER **do**
  Let the current position along the Ray be x^α.
  Let current time be t and current time step be dt.
  Move forward from this position by integrating the geodesic, to a new position along the ray y^α.
  if x^α and y^α are on opposite sides of the surface of any object in the scene **then**
    Find perpendicular distance, δ, to the object surface from current point.
    if $\delta < \epsilon$ **then**
      return radiance and normal at point on surface.
    else
      Reset current position on ray to x^α.
      $dt = dt/2$
    end if
  end if
end while
return no intersection (radiance returned is zero).

4 RESULTS AND DISCUSSIONS

In Figure 2, a black hole of Schwarzschild radius 1.6 units sits inside a refractive ball of refractive index 1.5 which has a radius of 16 units. We can see rings of darkness in the regions where the black hole directs light rays at an angle such that they don't refract out of the ball. Very notable is the dual image of the refractive ball against the blue background on the right hand side of the ball (shown in closeup). Those rays reflect off the reflecting ball on the left and then enter again into the refractive ball, where the black hole curves different rays from the same part of the scene into different regions of the image, thus allowing us to see multiple images of the same source.

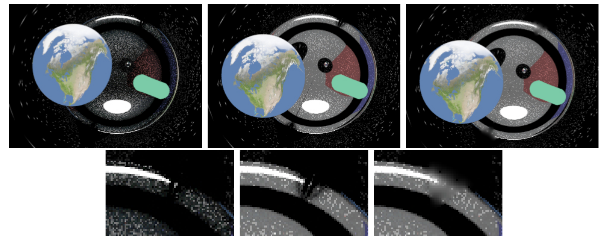

Figure 3: The top row shows the view from one end of the Ellis wormhole that is in outer space. The bottom row shows detail of what happens in a region of numerical instability with the three respective rendering methods used above.

In Figure 3, we show a view from a wormhole, with one end inside the room and the other end in outer space. A satellite (shown as the green cylinder) orbits the earth in outer space. The view of the room (top left) appears dim when seen through the wormhole end in outer space, because in the Monte Carlo path tracer very few ray samples make it through the wormhole back to the light source in the room. This can be corrected (top middle) by bidirectional path tracing in curved spacetime, caching the illumination from the light source to the walls in the room in light maps stored on the room walls. The wormhole geodesic solution is not well behaved near the poles. We terminate the rays near the poles to avoid numerical instability. These error regions can be filled (top right) by propagating the intensity information from nearby non-error pixels to the pixels that are in the error envelop.

4.1 Limitations and Future Work

Our algorithm is a brute force path tracing algorithm. We want to evaluate strategies like multiple importance sampling and tracking ray differentials in curved spacetime to increase its efficiency. Leveraging earlier work in non-linear raytracing, we also want to include the effects of participating media in curved spacetime. A current limitation of our renderer is that it cannot visualize dynamic scenes in curved spacetimes, as the animation trajectories of such movement would also curve, which we do not presently compute. Also of interest may be setting up space partitioning schemes in curved spacetime to accelerate such renders.

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