Scalable Visual Simulation of Ductile and Brittle Fracture

Avirup Mandal
avirupmandal@ee.iitb.ac.in
IIT Bombay
Mumbai, Maharashtra, India

Parag Chaudhuri
paragc@cse.iitb.ac.in
IIT Bombay
Mumbai, Maharashtra, India

Subhasis Chaudhuri
sc@ee.iitb.ac.in
IIT Bombay
Mumbai, Maharashtra, India

Figure 1: Our model produces the intricate fracture patterns that result from the tearing of a loaf of bread. The loaf model has around 620k tetrahedra.

ABSTRACT

Fracture of solid objects produces debris. Modelling the physics that produces the broken fragments from the original solid requires an increase in the number of degrees of freedom. This causes a huge increase in computational cost for FEM based methods used to model such phenomena. We present a graph-based FEM method that tackles this issue by relabeling the edges of the graph induced in a volumetric mesh, using a damage variable. We reformulate the system dynamics for this relabelled graph in order to simulate the fracture mechanics using FEM without an explosion in the computation cost. Our method therefore requires no remeshing of the volumetric mesh used for computation and this makes it very scalable to high-resolution meshes. We demonstrate that the method can simulate both brittle and ductile fracture.

KEYWORDS

fracture, remeshing-free, FEM, graph-based

ACM Reference Format:

Avirup Mandal, Parag Chaudhuri, and Subhasis Chaudhuri. 2021. Scalable Visual Simulation of Ductile and Brittle Fracture. In *Special Interest Group on Computer Graphics and Interactive Techniques Conference Posters (SIGGRAPH '21 Posters), August 09-13, 2021.* ACM, New York, NY, USA, 2 pages. https://doi.org/10.1145/3450618.3469152

1 MOTIVATION

We can observe ample examples of both brittle fracture like breaking of a glass tumbler, and ductile fracture like the tearing of bread, around us.

However, existing methods for visual simulation of fracture still suffer from certain limitations. E.g., the Finite Element Method (FEM) based approaches [O'Brien and Hodgins 1999] require remeshing to address the additional degrees of freedom (DOF) introduced by fracture, leading to generation of degenerate elements, instability and heavy computational cost. In remeshing-free techniques like eXtended Finite Element Method (XFEM) [Chitalu et al. 2020], the system matrix scales rapidly with the introduction of cracks. Peridynamics [Levine et al. 2015] methods deal with solving the integral equations of continuum mechanics, which is also computationally extremely expensive.

We propose a method that is free from remeshing, computationally light and does not scale the system matrix with an increase in the number of broken fragments.

2 METHOD

We propose a remeshing-free, graph-based FEM method for visual simulation of fracture in 3D. Our method is an enhancement of a 2D method from material science proposed by [Khodabakhshi et al. 2016] and works with both linear and non-linear strain energy. We start with a computational volumetric mesh, $\mathcal{M}_c$, made of tetrahedral elements. $\mathcal{M}_c$ induces a graph where vertices and edges of the mesh become the nodes and edges of the graph. We then project the stress of a tetrahedral element, Δ_e, of the mesh in the direction of edges of the graph formed by the edges of the tetrahedra.

$$\begin{aligned}\sigma^e_{mn} &= \sigma^e_{xx}\cos^2\phi_x + \sigma^e_{yy}\cos^2\phi_y + \sigma^e_{zz}\cos^2\phi_z \\ &\quad + \sigma^e_{xy}\cos\phi_x\cos\phi_y + \sigma^e_{xz}\cos\phi_x\cos\phi_z + \sigma^e_{yz}\cos\phi_y\cos\phi_z\end{aligned} \tag{1}$$

where σ^e_{mn} represents the normal stress along the edge formed by nodes m and n and $\sigma^e_{ij}, \forall i, j \in \{x, y, z\}$ are Cartesian components of Piola Kirchhoff stress. Similarly $\theta_k, \forall k \in \{x, y, z\}$ is the angle of the edge with k axis. If σ^e_{mn} exceeds a threshold, we mark the edge as damaged. A damaged edge is never repaired in subsequent simulation steps. Next, we recalculate the hyper-elastic strain energy of the element, Ψ^e, to include the effect of the damaged edge as

$$\Psi^e_{frac} = f(\phi_l)\Psi^e \tag{2}$$

where $f(\phi_l)$ denotes the update function based on damaged edge variable ϕ_l. This newly reformulated strain energy adds more freedom to the vertices of the broken edge for movement. Thus, if all the edges connecting a vertex get labelled as damaged it can move independently and $\mathcal{M}_c$ never needs to be remeshed. Additionally, using a kernel of support R_d, we can control the diffusion of cracks inside $\mathcal{M}_c$. Larger value of R_d produces more diffused

SIGGRAPH '21 Posters, August 09-13, 2021, Virtual Event, USA

ACM ISBN 978-1-4503-8371-4/21/08.
https://doi.org/10.1145/3450618.3469152

cracks (Figure 1) and vice versa (Figure 3). A visualization surface

Algorithm 1: Remeshing-Free Graph-based Fracture

Initialize FEM simulation;
while *True* **do**
  for *Each element in* $\mathcal{M}_c$ **do**
    Calculate stress along the edges σ_{mn};
    if $\sigma_{mn} > \sigma_{thres}$ **then**
      Label the edge as damaged;
      Remesh the surface of the corresponding $\mathcal{M}_v$;
    end
    Resolve all collisions with $\mathcal{M}_c$;
    Calculate impulse force due to collision;
    Add all external forces to the vertices of $\mathcal{M}_c$;
  end
  Build full system $[M]_{n_v \times n_v} [v]_{n_v \times 1} = [f]_{n_v \times 1}$;
  Solve for velocity vector $[v]_{n_v \times 1}$;
  for *Each vertex in* $\mathcal{M}_c$ *and* $\mathcal{M}_v$ **do**
    Update position vector by $[x]_{n_v \times 1} += \Delta t \cdot [v]_{n_v \times 1}$;
  end
end

mesh, $\mathcal{M}_v$, is maintained in addition to $\mathcal{M}_c$, and is the same as the outer surface of volumetric mesh initially. $\mathcal{M}_c$ needs to be split and the fracture surfaces have to be reconstructed for rendering the fracture. Remeshing $\mathcal{M}_c$ does not affect the $\mathcal{M}_v$. We present the complete pseudo code for our method in Algorithm 1. n_v is the total number of vertices of $\mathcal{M}_c$ and $[x]_{n_v \times 1}$ is the initial position vector.

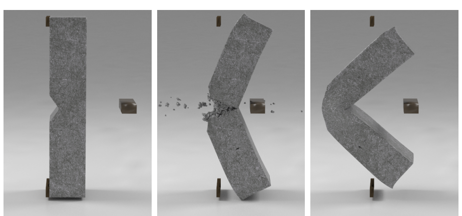

Figure 2: Charpy Impact Test: Configuration and results with low and high tensile strength steel specimens.

3 VALIDATION

We validate our method by comparing the results produced by our method to a benchmark structural mechanics laboratory experiment called the Charpy Impact test. We create a steel specimen of dimension $10mm \times 10mm \times 55mm$ with a 45° 'V' shaped groove in the middle of it (see left image of Figure 2). The specimen is hit by a pendulum of known mass and length. We simulate this experiment using our method, on specimens made of two different kinds of steel with different tensile strengths. As shown in Figure 2, the steel specimen with lower tensile strength (middle image) breaks into two pieces while the one with higher tensile strength (right image) splits partially and bends. This is very similar to the behaviour shown by the actual laboratory experiments.

4 EVALUATION

In Figure 1, we show a visualization of a loaf of bread being torn by applying a pulling force at its ends. The highly detailed and intricate fracture effects produced show the stability and scalability of our method for ductile fracture. In Figure 3, a solid ball hits an armadillo made of solid blue jade to generate large chunks of debris from a brittle fracture.

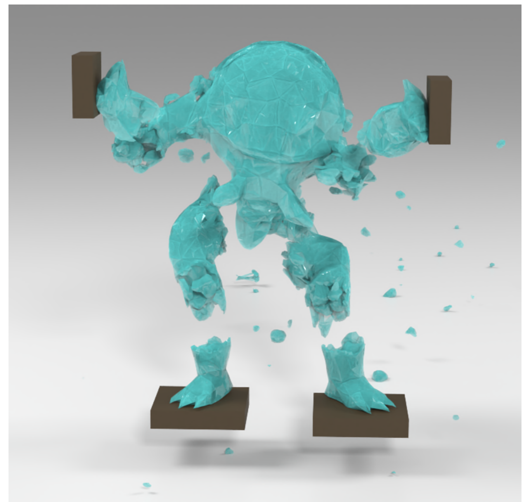

Figure 3: Fracture of an armadillo model made of solid blue jade.

Our method is much faster than the existing fracture methods, e.g., while a state of the art XFEM-based fracture simulation [Chitalu et al. 2020] requires 1539 sec to render a single frame for a 40k model, our model requires 291.7 sec to render a frame for the simulation of an extremely complex loaf model consisting of 620.6k tetrahedra. More details and results with extensive quantitative validation of our method can be found in [Mandal et al. 2021].

5 CONCLUSION

We present a novel graph-based remeshing-free FEM approach for ductile and brittle fracture. Even though our method is capable of producing particularly realistic fracture of a variety of materials at a high speed, it does have some limitations. Currently an edge of an element in the object mesh can split in maximum two parts, thus limiting the fracture of tetrahedral elements at maximum to four parts. We wish to extend our work to incorporate high number of fracture segments in a single element.

REFERENCES

F. M. Chitalu, Q. Miao, K. Subr, and T. Komura. 2020. Displacement-Correlated XFEM for Simulating Brittle Fracture. *Comp. Graph. Forum* 39, 2 (2020), 569–583.

P. Khodabakhshi, J. N. Reddy, and A. Srinivasa. 2016. GraFEA: a graph-based finite element approach for the study of damage and fracture in brittle materials. *Meccanica* 51 (2016), 3129 – 3147.

J. A. Levine, A. W. Bargteil, C. Corsi, J. Tessendorf, and R. Geist. 2015. A Peridynamic Perspective on Spring-Mass Fracture. In *Proc. of SCA'14*. 47–55.

A. Mandal, P. Chaudhuri, and S. Chaudhuri. 2021. Remeshing-Free Graph-Based Finite Element Method for Ductile and Brittle Fracture. arXiv:2103.14870

J. F. O'Brien and J. K. Hodgins. 1999. Graphical Modeling and Animation of Brittle Fracture. In *Proc. of SIGGRAPH '99*. 137–146.