Point Light Attenuation Without Singularity

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(a) Virtual Point Lights (no clamping) **(b)** Our Simplified Virtual Spherical Lights

Figure 1: *Virtual point lights with inverse-square attenuation causing illumination spikes that are eliminated with our attenuation function.*

ABSTRACT

Point lights with an inverse-square attenuation function are commonly used in computer graphics. We present an alternative formulation of point light attenuation that treats point lights as simplified forms of spherical lights. This eliminates the singularity of the inverse-square light attenuation function and makes it easier to work with point lights in practice. We also present how the typical ad hoc modifications of the inverse-square formula can be improved based on our formulation.

CCS CONCEPTS

• **Computing methodologies → Rendering**.

KEYWORDS

Point lights, light attenuation, virtual point lights, spherical lights

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1 INTRODUCTION

Point lights are commonplace in computer graphics. Though they do not represent realistic light sources, they are even used with physically-based rendering due to their simplicity. They are also used as virtual lights for approximating global illumination.

The inverse-square attenuation is the *correct* formulation for point lights, but its singularity at zero distance, where the light intensity goes to infinity, causes practical problems. Typical solutions use ad hoc attenuation functions that avoid the singularity for positions that are "too close to the light," which is scene-dependent.

We identify that the main source of the problem is not the attenuation function itself, but treating the light source as a point in the first place. Indeed, in most cases point lights are used for approximating small light sources with non-zero sizes.

We present an alternative derivation of light attenuation that treats point lights as a simplified form of spherical lights, entirely eliminating the singularity. Our formulation preserves the simplicity of point lights, but does provide a close approximation of near illumination from spherical lights.

2 POINT LIGHT ATTENUATION

The rendering equation for the outgoing (i.e. reflected) light $\mathbf{L}_o$ from a surface point in the direction ω_o is written as

$$\mathbf{L}_o(\omega_o) = \int_\Omega \mathbf{L}_i(\omega_i)\ f_r(\omega_o, \omega_i)\ \cos\theta\ \mathrm{d}\omega_i\ , \tag{1}$$

where $\mathbf{L}_i$ is the incoming light along the direction ω_i, f_r is the bidirectional reflectance distribution function (BRDF) of the material, $\cos\theta = \omega_i \cdot \mathbf{n}$ is the geometry term with surface normal $\mathbf{n}$, and Ω is the hemisphere of all incoming light directions. For a single point light with intensity $\mathbf{I}$, Equation 1 can be written as

$$\mathbf{L}_o(\omega_o) = \mathbf{I}\ V(\omega_i)\ f_a(d)\ f_r(\omega_o, \omega_i)\ \cos\theta\ , \tag{2}$$

where $V(\omega_i)$ is the binary visibility function (accounting for the point light shadow), d is the distance to the light, and $f_a(d) = 1/d^2$ is the attenuation function.

To eliminate the singularity of $f_a(d)$ at $d = 0$, we treat the light as a spherical Lambertian emitter with a small radius r. Then, we simplify Equation 1 into the same form as in Equation 2 with a

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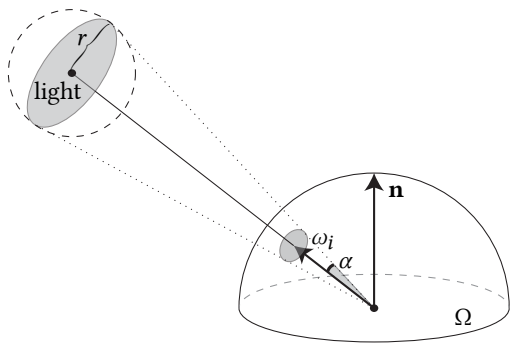

Figure 2: *Point light treated as a spherical light with radius r and the solid angle on the hemisphere Ω corresponding to its cross-section.*

different attenuation function. A spherical light emitting the same radiant power corresponds to $\mathbf{L}_i(\omega_\mathrm{i}) = \mathbf{I}/\pi r^2$ for all ω_i the light is visible. Thus, we can write Equation 1 as

$$\mathbf{L}_o(\omega_o) = \frac{\mathbf{I}}{\pi r^2} \int_\Omega V(\omega_i)\ f_r(\omega_o, \omega_i)\ \cos\theta\ \mathrm{d}\omega_i\ . \tag{3}$$

Treating the light source as a point light for the BRDF evaluation and the geometry term with a constant light direction ω_i, we get

$$\mathbf{L}_o(\omega_o) \approx \frac{\mathbf{I}}{\pi r^2}\ f_r(\omega_o, \omega_i)\ \cos\theta \int_\Omega V(\omega_i)\ \mathrm{d}\omega_i\ . \tag{4}$$

If the light is not shadowed, the integral of the visibility function is equal to the area of the solid angle through which the light is visible on the unit sphere (Figure 2). Assuming that the entire solid angle is contained within the hemisphere Ω, we can write

$$\int_\Omega V(\omega_i)\ \mathrm{d}\omega_i\ \approx 2\pi\,(1-\cos\alpha)\,V(\omega_i)\ . \tag{5}$$

Under these assumptions, $\mathbf{L}_o(\omega_o)$ can be written in the same form as in Equation 2 with a different attenuation function

$$f_a(d) = \frac{2\pi\,(1-\cos\alpha)}{\pi r^2}\ . \tag{6}$$

For a spherical light, $\cos\alpha$ is not defined for $d < r$. Instead, we approximate the light as a *disk light* aligned with the light direction ω_i, as shown in Figure 2, which produces identical illumination for $d \gg r$. Thus, we get our final formulation of light attenuation

$$f_a(d) = \frac{2}{r^2}\left(1 - \frac{d}{\sqrt{d^2+r^2}}\right)\ . \tag{7}$$

Our attenuation formulation in Equation 7 relies on the following simplifications to Lambertian spherical lights:

- The BRDF, the geometry term, and the light shadows are computed using a single light direction ω_i (Equation 4).
- The solid angle is assumed to be contained fully within the hemisphere Ω (Equation 5).
- The solid angle is approximated using the projected area of the light cross-section (i.e. disk light) (Equation 7).

Therefore, the attenuation formulation we describe above does not simply convert point lights to spherical lights. Instead, we treat point lights as point lights with a spherical radius, which is only used for computing their attenuation. Note that at the limit where r goes to zero, all of these simplifications are perfectly justified and, as expected, $f_a(d)$ approaches $1/d^2$.

3 COMMON AD HOC SOLUTION

It is typical to eliminate the singularity of the inverse-square attenuation function by simply adding a constant factor to the denominator of the fraction. The only practical difficulty with this approach is picking a reasonable constant value. We can remedy this using a user-specified light radius r, such that the attenuation function becomes

$$\tilde{f}_a(d) = \frac{1}{d^2 + \frac{r^2}{2}}\ , \tag{8}$$

which produces the same attenuation value at $d = 0$ as Equation 7. Note that $\tilde{f}_a$ is bounded by our attenuation function and inverse-square attenuation. However, unlike Equation 7, the physical meaning of this ad hoc solution is unclear.

4 EVALUATION

As expected, this formulation approaches to inverse-square attenuation when r goes to zero and for distant illumination ($d \gg r$). Therefore, in practice, our attenuation formulation produces virtually identical results to the inverse-square attenuation, unless the scene contains objects that are too close to the light source. In that case, we avoid the singularity at $d = 0$. Figure 3 compares our attenuation function, its simple approximation, and the correct inverse-square attenuation for point lights.

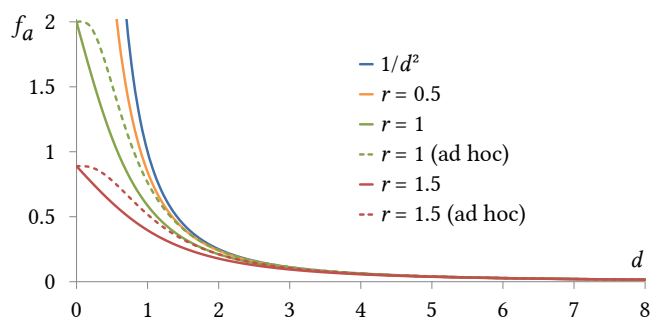

Figure 3: *Comparison of inverse-square attenuation, our formulation (Equation 7), and the ad hoc solution (Equation 8).*

While our formulation is designed for point lights manually placed in scenes with user-defined radius values, it can also be used to form a simplified version of virtual spherical lights [Hašan et al. 2009] with a much simpler formulation. Figure 1 shows such an example rendered using stochastic lightcuts [Yuksel 2019].

5 CONCLUSION

We have presented an alternative formulation that eliminates the singularity of the typical inverse-square attenuation function by simply adding a non-zero radius parameter to a point light. While this conceptually forms a spherical light, it does not closely approximate spherical lights, particularly for short distances.

REFERENCES

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