Underwater bubbles and coupling

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ABSTRACT

We present an approach to simulating underwater bubbles. Our method is sparse in that it only simulates a thin band of water around the region of interest allowing us to achieve high resolutions in turbulent scenarios. We use a hybrid bubble representation consisting of two parts. The *hero* counterpart utilizes an incompressible two-phase Navier-Stokes solve on an Eulerian grid with air phase also represented via FLIP/APIC particles to facilitate volume conservation and accurate interface tracking. The *diffuse* counterpart captures sub-grid bubble motion not "seen" by the Eulerian grid. We represent those as particles and develop a novel scheme for coupling them with the bulk fluid. The coupling scheme is not limited to sub-grid bubbles and may be applied to other thin/porous objects such as sand, hair, and cloth.

CCS CONCEPTS

• **Computing methodologies → Physical simulation**.

KEYWORDS

physical simulation, bubbles, coupling

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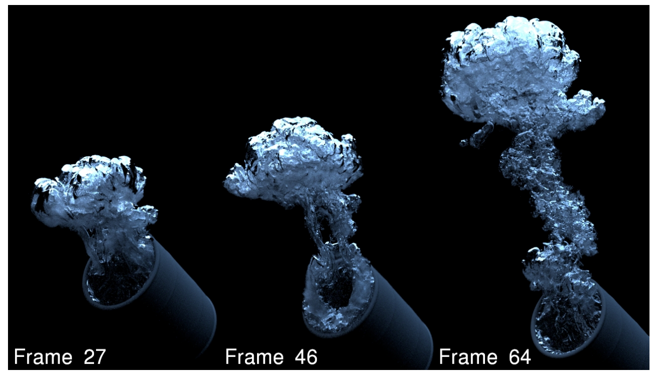

Figure 1: A large (~250 liters) barrel filled with air overturning under water is simulated with our method. ©Weta Digital Ltd 2020.

1 INTRODUCTION

Underwater bubbles are fascinating. Bigger volumes of air exhibit turbulent, almost explosive behavior as they shatter into mid-size, more stable pockets and myriads of tiny ones forming foggy aerated regions. The large density ratio between water and air (1000:1) is responsible for this beautiful violent dynamics, and is also a reason why bubbles are so difficult to simulate on a computer.

A number of papers have considered simulating underwater bubbles. [Goldade and Batty 2017] adopt a FLIP fluid simulator to represent each air pocket as a volume-conserving void with fixed pressure. While able to recreate realistic gargling water effects the method does not capture subtle bubble details. [Boyd and Bridson 2012] use FLIP to discretize both water and air and perform a two-phase incompressible solve. Bubbles smaller than a grid voxel size are typically represented as a separate particle system. [Kim et al. 2010] passively advect those particles with the bulk fluid and use them to adjust effective density of water, leading to naturalistic buoyancy effects. They employ a stochastic solver for additional sub-voxel motion. [Patkar et al. 2013] use an Eulerian two-phase approach for simulating bubbles larger than the grid voxel size and passively advected particles for tracking bubbles smaller than the grid voxel size. They combine the two in a single linear solve which also handles compressibility.

2 OUR APPROACH

We disregard compressibility for efficiency reasons and adopt a two-phase incompressible ghost-fluid Eulerian solve for our *hero* (larger than the voxel size) bubbles, similar to [Boyd and Bridson 2012]. Unlike them however, we use FLIP/APIC particles to only track the air phase and recover the interface, while discretizing water as purely Eulerian in a narrow band around the bubbles. As we did not want the water to have an apparent sliding effect with respect to invisible boundaries and also to avoid dealing with null-modes in the Poisson pressure solve, we enforce hydrostatic pressure boundary condition, as opposed to a flux velocity boundary condition, on the outside of the narrow band $p(h) = \rho_w g h$, where h is the evaluation height. An example of a hero bubble simulation is shown in Figure 1.

We wanted our *diffuse* (smaller than the voxel size) bubbles to accurately capture sub-grid dynamics, unlike previous methods that would use them as trackersto modify the effective water den-

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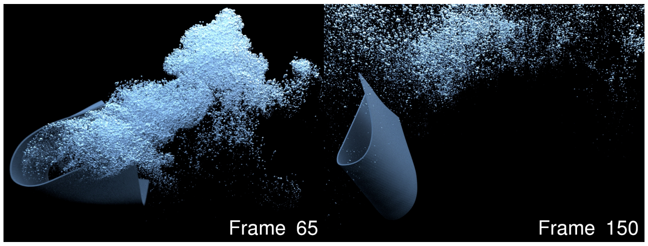

Figure 2: Diffuse bubbles and cloth are coupled with FLIP water through drag and buoyancy. ©Weta Digital Ltd 2020.

sity/expansion. We assign them velocities of their own and couple with the bulk fluid through buoyancy and drag forces. In the limit of infinite drag, our approach is equivalent to "passive advection + density adjust", a so-called *Boussinesq approximation.*

3 DIFFUSE BUBBLES AND COUPLING

Governing equations. To model interactions between the diffuse bubble particles and the bulk fluid we adopt a continuum approach. Let ϕ_b and $\phi_w = 1 - \phi_b$ denote the volume fractions of bubbles and water respectively. Following [Anderson and Jackson 1967; Daviet and Bertails-Descoubes 2017], we write the internal pressure of the water and bubbles as fractions of the pore pressure p, that is $\phi_b p$ and $\phi_w p$, respectively. The force per unit volume exerted by the water onto the bubbles is the sum of a generalized buoyancy contribution, $\boldsymbol{f}^{\text{buo}}_{w\to b} = \phi_w \nabla(\phi_b p) - \phi_b \nabla(\phi_w p) = p\nabla\phi_b$, and a drag term, $\boldsymbol{f}^{\text{drag}}_{w\to b} = \xi(|\boldsymbol{u}_w - \boldsymbol{u}_b|)(\boldsymbol{u}_w - \boldsymbol{u}_b)$, with ξ a possibly non-uniform drag coefficient. The conservation of momentum equations for the two phases thus read

$$\phi_w\rho_w\frac{D\boldsymbol{u}_w}{Dt} - \phi_w\rho_w\boldsymbol{g} + \boldsymbol{f}^{\text{drag}}_{w\to b} = -\boldsymbol{f}^{\text{buo}}_{w\to b} - \nabla(\phi_w p) \equiv -\phi_w\nabla p,$$
$$\phi_b\rho_b\frac{D\boldsymbol{u}_b}{Dt} - \phi_b\rho_b\boldsymbol{g} - \boldsymbol{f}^{\text{drag}}_{w\to b} = \boldsymbol{f}^{\text{buo}}_{w\to b} - \nabla(\phi_b p) \equiv -\phi_b\nabla p.$$

Assuming a non-zero water fraction and integrating the bubbles conservation equation over the volume V_p of a particle yields

$$\rho_w\frac{D\boldsymbol{u}_w}{Dt} = \rho_w\boldsymbol{g} - \frac{1}{\phi_w}\boldsymbol{f}^{\text{drag}}_{w\to b} - \nabla p \qquad \text{(water)}$$
$$m_p\frac{\mathrm{d}\boldsymbol{u}_p}{\mathrm{d}t} = m_p\boldsymbol{g} + \frac{V_p}{\phi_b}\boldsymbol{f}^{\text{drag}}_{w\to b} - V_p\nabla p \qquad \text{(bubble particles)}$$

where m_p and $\boldsymbol{u}_p$ denote the mass and velocity of the bubble particle, respectively. The system is closed by enforcing incompressibility of the mixture, $\nabla\cdot[\phi_w\boldsymbol{u}_w + \phi_b\boldsymbol{u}_b] = 0$. By adding interparticle forces/constraints the model is naturally extended to handle thin/porous solids such as hair, plants and cloth, see Figure 2.

Discretization. To benefit from efficient solvers for each object we couple diffuse bubbles and the bulk fluid weakly. This becomes especially useful for more complicated submerged materials such as cloth and hair as they exhibit non-trivial elastic responses. Within each Newton step of the solver we perform a Poisson projection to obtain fluid velocity and pore pressure assuming prescribed velocities of submerged materials, and then solve for the latter with fluid velocity and pressure fixed. The diffuse bubbles are modeled as Lagrangian particles interacting with the Eulerian bulk fluid through buoyancy and drag, and additional splatting/interpolation is required to transfer particle data to/from the Eulerian grid.

The drag force is calculated on the particles from the surrounding fluid velocity. It is rasterized to the grid with a minus sign, and added as a force term to the bulk fluid. Typically, a linear drag model is assumed when fluid flow around bubbles can be considered laminar, however we have found that to not be the case in our turbulent scenarios, and a quadratic drag force term was used to create more realistic dynamics. Due to the non-linearity we have found that an implicit treatment of the rasterized drag within the Poisson pressure solve was important to achieve good convergence results.

4 DISCUSSION

Extensions. The method can be employed as post-processing technique on top of an existing fluid simulation with no bubbles. Using the pressure values from the existing simulation as a boundary condition on our sparse fluid domain, and "air bubble entrainment" metric from [Gualtieri et al. 2008] to emit both hero and diffuse bubbles we were able to achieve believable results.

Limitations and future work. Sparsity limits the hero bubble technique to working under water only, and bubbles breaking the surface may be an interesting future work exploring alternative boundary conditions; in our post-processing application we simply removed particles crossing the surface of the primary simulation. Drag force coefficient needed to be tuned to achieve consistency between hero and diffuse representations. Our diffuse bubbles do not merge, and we would like to adopt methods such as [Jones and Southern 2017].

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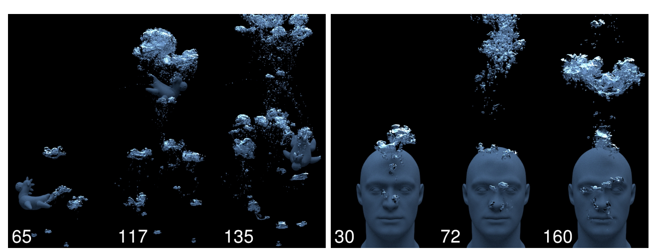

Figure 3: Frames of a toy moving through a bubble flow (left) and a person breathing underwater (right). ©Weta Digital Ltd 2020.