Shape Targeting: A Versatile Active Elasticity Constitutive Model

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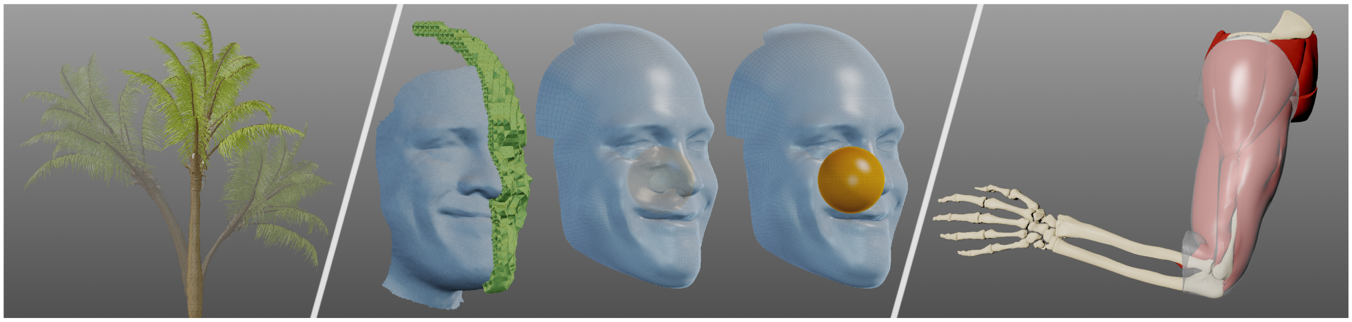

Figure 1: Shape targeting in action. Left: palm tree animated by procedurally generated shape targets. Center: Guided simulation. A facial scan is driving the primary simulation to set the shape targets. Then shape targets are used recreates the pose and incorporate collisions. Right: muscle flexing simulated by computing shape targets from fiber directions and activations.

ABSTRACT

The recent "Phace" facial modeling and animation framework [Ichim et al. 2017] introduced a specific formulation of an elastic energy potential that induces mesh elements to approach certain prescribed shapes, modulo rotations. This target shape is defined for each element as an input parameter, and is a multi-dimensional analogue of activation parameters in fiber-based anisotropic muscle models. We argue that the constitutive law suggested by this energy formulation warrants consideration as a highly versatile and practical model of active elastic materials, and could rightfully be regarded as a "baseline" parametric description of active elasticity, in the same fashion that corotational elasticity has largely established itself as the prototypical rotation-invariant model of isotropic elasticity. We present a formulation of this constitutive model in the spirit and style of Finite Element Methods for continuum mechanics, complete with closed form expressions for strain tensors and exact force derivatives for use in implicit and quasistatic schemes. We demonstrate the versatility of the model through various examples in which active elements are employed.

CCS CONCEPTS

• **Computing methodologies** → **Physical simulation**; *Procedural animation.*

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KEYWORDS

simulation, elasticity, active deformations, animation control

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1 INTRODUCTION

Elastic deformations driven by internal factors represent an important aspect of physically based animations. In these scenarios the discrete mesh elements of a volumetric body induce internal forces that drive them, not just in response to external stimuli or constraints, but as an intrinsic action. Such active deformations can be based on biomechanics by modelling muscle contractions, may be employed to guide the simulation towards a pre-described shape, or they can be purely artist driven. Ichim et al. [2017] introduced an elastic energy potential to model the active and passive tissues of facial animations. Their system prescribes local deformations for each tetrahedral element, and the simulation drives the elements towards these target deformations.

In this talk, we demonstrate the versatility of this paradigm in modeling a much broader class of active elastic models. We facilitate this broader applicability by casting the formulation of Ichim et al. [2017] in the language and solver framework of traditional continuum mechanics constitutive models, such as Corotated and Neo-hookean Elasticity that are popular in graphics applications, but with the potential to encode active internal deformations.

2 TECHNICAL APPROACH

Constitutive model. We expand upon the deformation energy introduced by Ichim et al. [2017] to re-cast it in the traditional mathematical framework of an elastic constitutive model via the following equations:

$$\Psi(\mathbf{F}) = \min_{\mathbf{R}\in SO(3)} \mu\|\mathbf{F}-\mathbf{R}\mathbf{S}_t\|_F^2 \quad (1)$$

$$= \|\mathbf{F}-\mathbf{R}_*\mathbf{S}_t\|_F \quad (2)$$

$$\mathbf{P}(\mathbf{F}) = 2\mu(\mathbf{F}-\mathbf{R}_*\mathbf{S}_t) \quad (3)$$

$$\delta\mathbf{P}[\mathbf{F};\delta\mathbf{F}] = 2\mu(\delta\mathbf{F}-\delta\mathbf{R}_*[\mathbf{F};\delta\mathbf{F}]\mathbf{S}_t) \quad (4)$$

$$\delta\mathbf{R}_*[\mathbf{F};\delta\mathbf{F}] = \mathbf{U}_*\{\mathcal{E}:[\mathbf{K}^{-1}\mathcal{E}^T:(\mathbf{U}_*^T\delta\mathbf{F}\mathbf{S}_t\mathbf{V}_*)]\}\mathbf{V}_*^T \quad (5)$$

$$\mathbf{K} = \mathrm{tr}(\Sigma_*)\mathbf{I}-\Sigma_* \quad (6)$$

Ichim et al. [2017] proposed a deformation energy (eq. 1) that used a symmetric 3×3 "activation" matrix $\mathbf{S}_t$ as a descriptor of the target shape that a tetrahedral element is encouraged to assume. By minimizing over all rotation matrices, arbitrary rotations were factored away from $\mathbf{S}_t$, allowing it to encode pure shape change; this also allows using a local-global iteration, in the spirit of Projective Dynamics [Bouaziz et al. 2014] to produce an animation under quasistatic or implicit integration.

We start our restatement of this formulation by replacing the minimization formulation of the problem with a closed form expression (eq. 2), where the minimizer has been explicitly noted as $\mathbf{R}_*$; recognizing this as an instance of the Orthogonal Procrustes problem, we can compute this minimizer as $\mathbf{R}_*(\mathbf{F};\mathbf{S}_t)=\mathbf{U}_*\mathbf{V}_*^T$, where the matrices used in this expression are taken from the SVD of $\mathbf{F}\mathbf{S}_t=\mathbf{U}_*\Sigma_*\mathbf{V}_*^T$. The notation $\mathbf{R}_*(\mathbf{F};\mathbf{S}_t)$ is intended to make explicit that $\mathbf{R}$ is a function of both the current deformation gradient, as well as the "shape target" $\mathbf{S}_t$ – a user-specified parameter.

Treating this energy formula as a starting point, we have derived the exact expression for the first Piola stress (eq. 3), using which we can directly compute forces in tetrahedral or hexahedral meshes. The exact differential of the stress can similarly be computed (eq. 4) using an expression for the differential of $\mathbf{R}_*$ (eqs. 5,6) which can be obtained using a similar proof pattern as seen in Corotated elasticity [McAdams et al. 2011]; here $\mathcal{E}$ denotes the alternating tensor (or Levi-Civita symbol). These results make it straightforward to employ full-fledged Newton-style second order methods (in addition to the option of Projective Dynamics) for the solution of quasistatic or implicit time integration problems. Lastly, it is possible to perform an explicit projection of the elemental stiffness matrix to its positive semi-definite component (see supplemental document) to use within the context of modified Newton schemes that employ solvers for the linearized equations that require positive definiteness (e.g. Conjugate Gradients).

Related methods. To model the action of active elements Coros et al. [2012] decomposed their rest mesh into disconnected elements, and modified those to incite motion in what they call "Incompatible Shape Interpolation." While the elastic forces still drive the elements to their rest shapes, the repeated update of these incites the active deformations. These two works are closest in spirit to the notion of shape targeting. It is notable however that adapting the rest state to encode active shape change comes with the risk of poor conditioning of the stiffness matrix if the rest shape is made to consist of nearly flattened elements, and this paradigm becomes incompatible with the use of Projective Dynamics as a solver, while also complicating the application of second-order Newton schemes. Finally, multiplicative plasticity [Klár et al. 2016] has been used in engineering and graphics to capture the permanent shape change of the elements. It works by attributing a portion of the current deformation of an element to change of its rest pose. Similar to rest-shape adaptation, this formulation can be problematic in the case where the "plastic" factor of the deformation gradient becomes degenerate or inverted.

3 PRACTICAL DEMONSTRATIONS

We highlight three use scenarios of shape targeting in simple demonstrations inspired by common uses of active models in production tasks. Since the constitutive model is a basic building block of more complex system (potentially incorporating collision processing, inverse problems, or embellishments to aid art-directability) these are mere proofs-of-concept rather than demonstrations of end-to-end pipelines. Either full-Newton schemes or Projective Dynamics local/global iterations are viable solvers for the examples presented.

Artistic control. Figure 1 (left) illustrates the use case of artist driven simulations. The palm tree has been embedded in a tetrahedral mesh, then the artist procedurally generated the $\mathbf{S}_t$ matrices for each element per frame. The individual deformations are not compatible – it would be impossible to have every element in their target shape without breaking the mesh. It is the goal of the solve itself to arrive to a state that matches these targets as close as possible, incorporating any additional external forces, if present.

Guided simulation. To create the guided performance of the face in Figure 1 we employed a two stage approach. First, the simulation is performed using the vertices of a scanned performance as attachments to simulated volume, and with all shape targets set to identity. Once a frame is solved, the symmetric part of each element's deformation gradient is stored. In the second stage, these stored matrices are used as shape targets. Using only the shape targets the simulation can recreate the performance without the use of the original scans, optionally including extra physical interactions.

Fiber-based muscles. Figure 1 (right) is an example use of shape targets in a fiber-based muscle model. The entire upper arm is simulated as a single volume. The area of influence of each muscle is represented by their surface geometries. Curves represent the fiber directions, and activation values define the intensity of contractions. Each tetrahedral element can receive full or fractional contribution from a single or multiple muscles. The shape targets are computed from the combined effect of the influencing muscles.

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