# An Interactive Course on Fractals and Chaos

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This paper describes a course that teaches fractals and chaos via significant student interaction with computer software. The course was developed and taught jointly by the authors (a computer science faculty member with expertise in computer graphics and a mathematics faculty member with expertise in chaotic dynamical systems). Exercises for students to investigate properties of chaotic dynamical systems were developed for computer algebra systems (Maple and Mathematica). Interactive computer-graphics-based software developed by the instructors provided the ability for students to investigate fractals. Lecture material provided the theory on fractals and chaos and the relationship between them. Project work allowed the students to reinforce their understanding of the concepts. Students learned both mathematical and computer graphics concepts in the course, and could receive course credit either in mathematics or computer science.

#### Introduction

Ever since Mandelbrot popularized fractals, students of mathematics have been intrigued by the theory behind them. Likewise, students of computer science, and especially computer graphics, have been engrossed in the beauty of fractal images. It wasn't long before students requested courses to learn about fractals and chaos, from both mathematical and computer graphics perspectives.

For several years, each of us has been teaching special topics courses on this material within our respective departments. The computer science course covered fractals from a computer graphics perspective but included some mathematics for theory. On the other hand, the mathematics course was mostly theoretical, using some simple computer graphics programs to demonstrate fractals as examples of chaotic dynamical systems. In our discussions, it became apparent that the two courses could benefit from each other. Thus, we decided to develop a single course covering fractals and chaotic dynamical systems that we would teach together. The course would be cross listed in both departments, and students could earn either computer science or mathematics credit for the course.

#### The Course

We each received a one-course release in the winter term of the 1996-1997 school year to develop the syllabus and class materials for the new course, which was taught for the first time in the spring 1997 quarter. As a unifying theme, we chose to concentrate on dynamics of the quadratic map (z { $z^2$ + c). This enabled us to learn about graphical analysis of maps, controlling chaos, Cantor sets, fractal dimensions, the Mandelbrot set, and Julia sets. To add breadth to the course, iterated function systems, random fractals, and other maps were studied as well. We chose "Chaos and Fractals: New Frontiers of Science"[1] as the reference book for the course because of its readability and broad coverage of the discipline.

Our primary goal was to get our students motivated to learn about fractals and chaotic dynamical systems. Since most of the topics are motivated by computer graphics, we developed software for the students to help them understand the course material. This included demonstration software (for observing and exploring properties of fractals and chaos) and software for students to modify as part of an assignment. In addition, we developed explorations with computer algebra systems (both Maple and Mathematica) so that our students could understand the underlying computations involved in creating the fractals. Since the course met in a classroom equipped with computer workstations, these computer algebra explorations could be done during class time in conjunction with the corresponding lecture material.

To allow the students to explore the Mandelbrot set and Julia sets and the relationship between them, we developed an interactive software product called MandelbrotExplore (written in C using OpenGL on SGI workstations). The program displays four windows on the screen (see Figure 1). In the upper left window, the Mandelbrot set initially appears. Students can, through use of the mouse, select a region of the Mandelbrot set to zoom in on and explore. By clicking on a point in the Mandelbrot set window, the Julia set for that point is generated and displayed in the upper right window. Students can zoom in on regions of the Julia set as well. Clicking on a point in the Julia set window generates the orbit for that point and displays the orbit in the lower left window on the screen. The lower right window is intended for showing numerical values for orbits inside the Julia set window, although it is not yet functional. (Source code, instructions on how to compile and use the software, and additional example screen shots can be found on the CD-ROM.) In addition to being a powerful research tool, the program provides computer science students with a complex (no pun intended!) example of how to produce fractals.

It is crucial to have a good comprehension of bifurcations to understand the Feigenbaum diagram as well as the Mandelbrot and Julia sets. To build this comprehension, we spent a fair amount of time exploring bifurcations with a computer algebra system. In addition to computing bifurcation values, the computer algebra system enabled students to find formulas for and graph the curves in the Feigenbaum diagrams.

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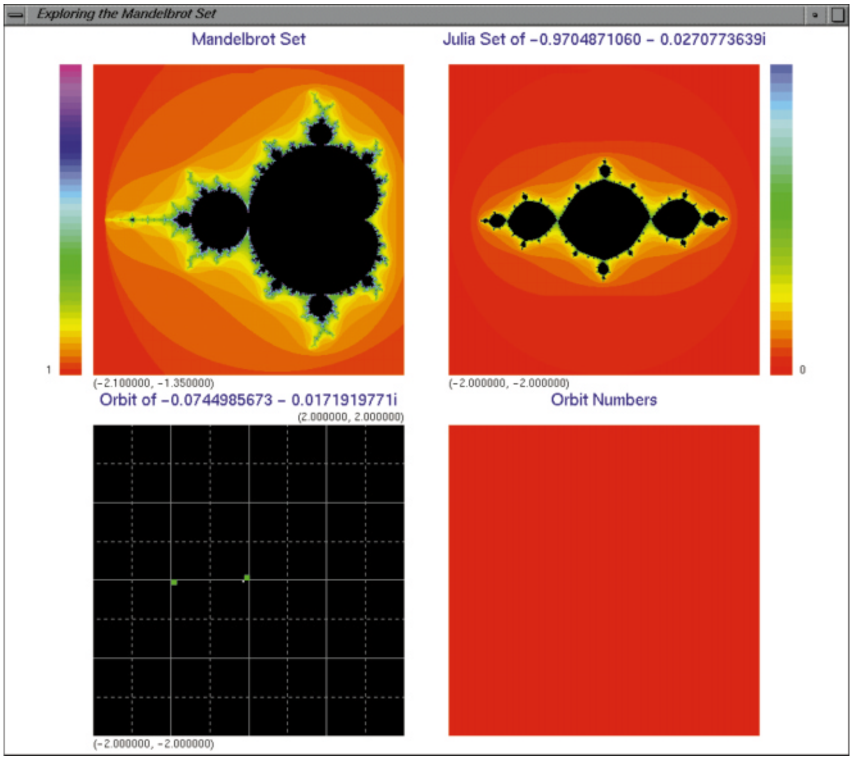


Figure 1 Screen image of the MandelbrotExplore program, an interactive software product that allows the user to explore the Mandelbrot set and Julia sets and the relationship between them.

When studying complex maps, students could repeat numerical computation of complex-valued orbits, and even determine the equation of the largest bulbs of the Mandelbrot set for the quadratic map, as shown in Figure 2. The computer algebra system was also used to understand the action of affine maps to help understand how iterated function systems worked. We also spent time learning how to control the chaotic quadratic map. Figure 3 shows a Maple worksheet in which the unstable period two orbit of the chaotic quadratic map is controlled by the OGY technique.[2]

**Results**

Enrollment in the first offering of the course was 19 students, consisting of computer science majors, mathematics majors, and a few engineering majors. Students were motivated to learn several major topics in the course, including:

- Complex algebra (through Mandelbrot and Julia Sets)
- Computer graphics
- Iteration (through dynamical systems and feedback)
- Bifurcation theory (through the Feigenbaum diagram and Mandelbrot Set)
- Cantor sets and countability
- Probability (through Brownian motion)

Students especially liked the week spent on controlling chaos. The computer algebra system was especially useful for helping students understand the underlying computations to control chaos. Our time and energy was devoted to the Mandelbrot set for the quadratic maps, but once students realized that there are other Mandelbrot sets (and subsequent families of Julia sets), the graphics-minded students seemed especially interested in working beyond the course to discover the shapes of Mandelbrot sets for other maps.

**Conclusion**

The course is now being offered annually as a joint elective within both the mathematics and computer science departments. Enrollment for the spring 1998 term is 19 students

**References**

1 Peitgen, Heinz-Otto; Jürgens, Hartmut; and Saupe, Dietmar. Chaos and Fractals: New Frontiers of Science. Springer-Verlag, 1992.

2 Ott, Edward; Grebogi, Celso; and Yorke, James A. Controlling Chaos. Phys. Rev. Lett. 64:1196-1199, 1990.

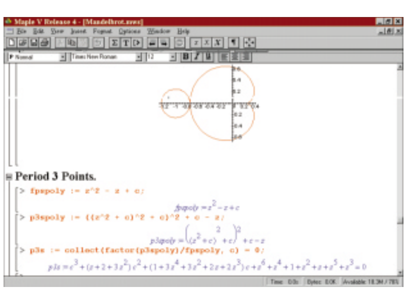


Figure 2
A Maple worksheet showing the equations and graphs for the largest bulbs of the Mandelbrot set.

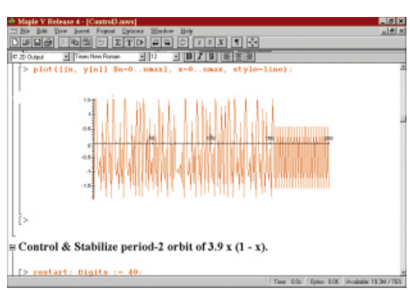


Figure 3
A Maple worksheet showing the unstable period two orbit of the chaotic quadratic map controlled by the OGY technique.