Improving Regularity of the Centoridal Voronoi Tessellation

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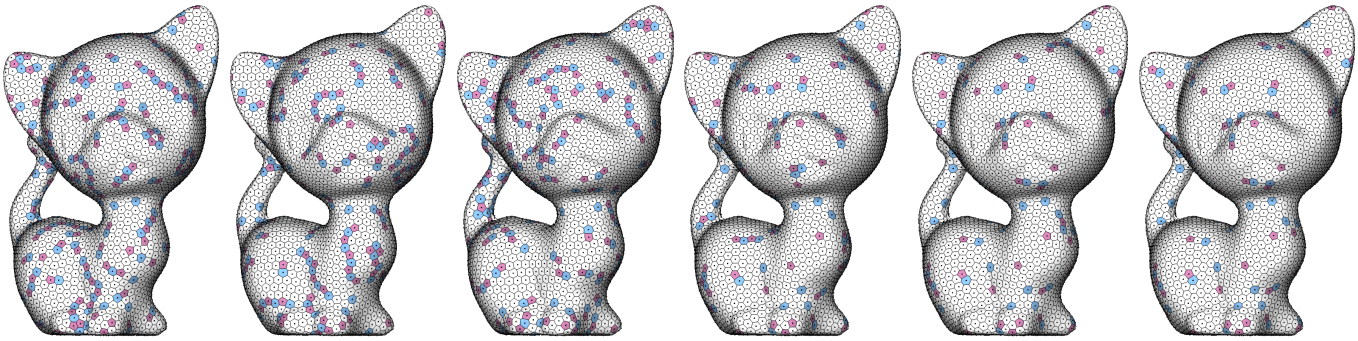

(a) $v_6\% = 80.99\%$ (b) $v_6\% = 83.33\%$ (c) $v_6\% = 85.26\%$ (d) $v_6\% = 90.28\%$ (e) $v_6\% = 92.48\%$ (f) $v_6\% = 92.95\%$

Figure 1: A comparison of different approaches for computing CVT on the Kitten Model with 3000 vertices. From left to right: CVT using quasi-Newton solver [Liu et al. 2009] (90 iterations), Monte-Carlo optimization [Lu et al. 2012] (1500 iterations), Hierarchal approach [Wang et al. 2016] (90 iterations), and Our method with 90, 180, and 270 iterations. Magenta color indicates valence 5 and blue color indicates valence 7. The others are regular cells with valence 6.

ABSTRACT

We present a novel method for valence optimization of the Centroidal Voronoi Tessellation (CVT). We first identify three commonly appeared atomic configurations of local irregular Voronoi cells, and then design specific atomic operations for each configuration to improve the regularity within the CVT framework.

CCS CONCEPTS

• **Computing methodologies** → **Computer graphics**;

KEYWORDS

valence optimization, centroidal Voronoi tessellation, regularity

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1 INTRODUCTION

Centroidal Voronoi Tessellation (CVT) is a special type of Voronoi diagram that requires each generator coincide with the centroid of its corresponding Voronoi cell, which has many applications especially in mesh generation. A CVT can be computed by minimizing the related energy function in Eqn. 1 [Du et al. 1999].

$$E_{CVT}(\mathbf{X}) = \sum_{i=1}^{n} \int_{\Omega_{i|\mathcal{M}}} \rho(\mathbf{x}) \|\mathbf{x} - \mathbf{x}_i\|^2 d\sigma, \tag{1}$$

where $\mathcal{M}$ is the sampling domain (mesh surface in our setting), Ω_i is the 3D Voronoi cell of $\mathbf{x}_i$, $\Omega_{i|\mathcal{M}} = \Omega_i \bigcap \mathcal{M}$ is the restricted Voronoi cell on the sampling domain, $\mathbf{X} = \{\mathbf{x}_i\}_{i=1}^n$ is the set of generators (or vertices). However, due to high non-convexity of the CVT energy, it is always trapped into local minima during the optimization process. It has been shown in previous work that the higher the regularity of the point distribution, the lower the CVT energy [Wang et al. 2016]. Here, the regularity indicates the number of neighboring Voronoi cells of each cell. Valence six means regular and others are irregular.

There exist several approaches for improving the regularity of CVTs. For example, the Newton-like solver [Liu et al. 2009] converges faster than standard Lloyd's algorithm [Lloyd 1982], hence results in more regular patterns. Alternatively, the convergence can be improved by cell wise operations, such as splitting and merging [Deussen et al. 2017]. Other works either using global Monte-Carlo optimization [Lu et al. 2012], or designing better strategy for initialization and hierarchical optimization [Wang et al. 2016] to improve the regularity.

In this poster, we present a new approach to improve the regularity of the CVT on mesh surfaces. The main contribution of our approach is that we carefully identified three configurations of neighboring Voronoi cells, and designed new atomic operation for each configuration to improve the regularity. Combined with existing fast CVT computation framework [Liu et al. 2009], we can

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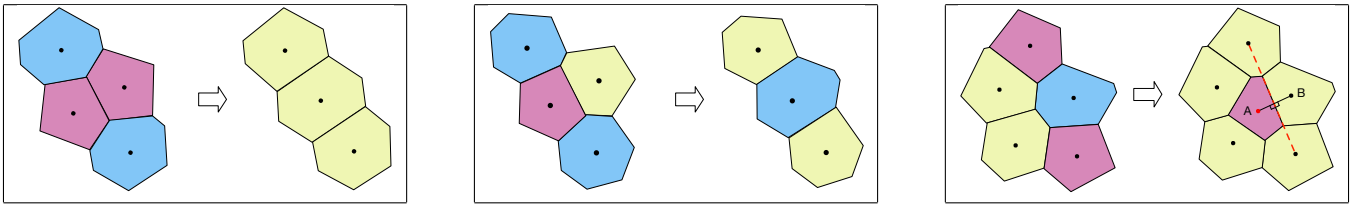

Figure 2: Three atomic operations in our method. Left: v_{5577}, middle: v_{757} and right: v_{575}. The new vertex *A* is inserted such that *A* and *B* are equidistant from the red line segment.

further improve the regularity of CVT compared with state-of-the-art techniques [Lu et al. 2012; Wang et al. 2016], see Fig. 1 for an example.

2 OUR APPROACH

We observed that valence 5 (v_5) and valence 7 (v_7) cells always appear together in a local region of an optimized CVT, surrounded by regular valence 6 (v_6) cells. By analysing these irregular patterns, we identify three configurations that appear frequently, i.e., two pairs of v_5 and v_7 cells, a chain of v_5, v_7, and v_5, and a chain of v_7, v_5, and v_7 cells. We call them v_{5577}, v_{757}, and v_{575}, respectively, as illustrated in Fig. 2. Inspired by the valence editing operation presented in [Li et al. 2010], we then design three atomic operations w.r.t. these configurations, i.e., (1) merging the v_5 pair of v_{5577} pattern results three v_6 cells, by carefully selecting the position of the new generator (Fig. 2(left)); (2) merging the v_5 cell and the common v_6 neighbor cell of a v_{757} pattern results a single v_7 cell (Fig. 2(middle)); (3) splitting the v_7 cell of a v_{575} pattern results a single v_5 cell (Fig. 2(right)). Note that our operations are different form [Li et al. 2010] since we aim at introducing more regular cells.

During the CVT optimization process (can be Lloyd [Lloyd 1982], Newton-like [Liu et al. 2009], or split and merge [Deussen et al. 2017]), we keep detecting such configurations and applying the same amounts of splitting and merging operations. The irregular cells can be reduced efficiently without sacrificing the performance.

3 EXPERIMENTAL RESULTS

We implemented our algorithm in the open-source platform *Geogram* (https://gforge.inria.fr/projects/geogram). Fig. 1 shows a comparison of our method with the state-of-the-art approaches. Our method exhibits higher regularity (see Fig. 3 for statistics). In our tests, given an initial sampling, we first run 3 ∼ 5 Lloyd iterations,

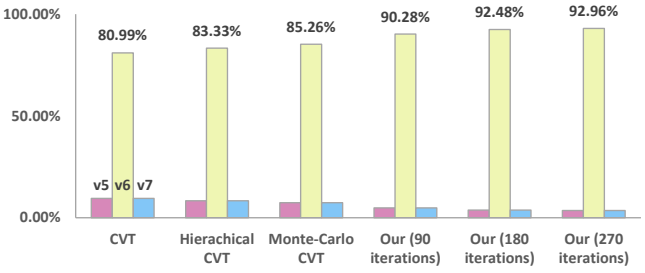

Figure 3: Histogram of the valences for Kitten example shown in Fig. 1.

and then switch to quasi-Newton iterations and apply our atomic operations every 30 iterations. Fig. 4 shows the intermediate results of our algorithm on the Botijo model. The final results of our method contains mostly isolated v_5, v_7 cells or v_{57} pairs. We plan to design new atomic operations to handle such configurations.

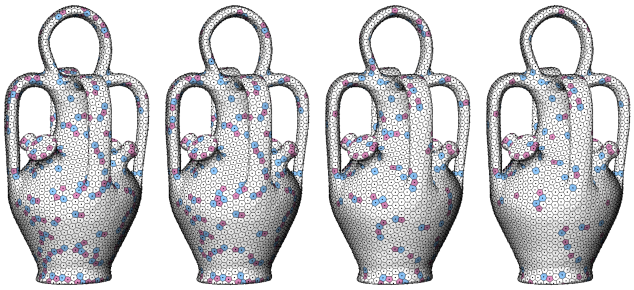

Figure 4: Intermediate results of the Botijo model. From Left to right: Initial CVT, results after applying atomic operations v_{5577}, v_{757} and v_{575}. The percentage of v_6 vertices are: 80.06%, 83.16%, 86.34%, and 88.22%.

4 CONCLUSION AND FUTURE WORK

We propose new atomic operations for improving the regularity of the CVT. Our initial experiments demonstrate that the proposed method works well for uniform sampling, and exhibits higher regularity than existing approaches. Next, we plan to extend this work to adaptive sampling and surface remeshing.

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