Curved Support Structures and Meshes with Spherical Vertex Stars

Martin Kilian
Vienna University of Technology

Hui Wang
Vienna University of Technology
Dalian University of Technology

Eike Schling
Technical University of Munich

Jonas Schikore
Technical University of Munich

Helmut Pottmann
Vienna University of Technology

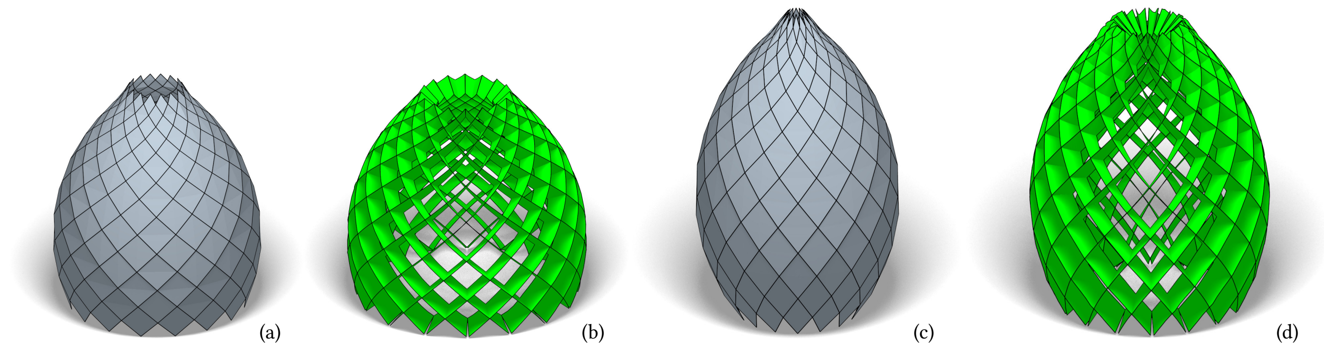

Figure 1: (a) A mesh with spherical vertex stars, computed from an unduloid surface. All vertex spheres are of the same radius. (b) The associated support structure consisting of developable strips with circular unrolling. (c) Deformed mesh, preserving sphere radii and edge lengths. (d) Associated support structure. Unrolled strips are identical to those shown in (b).

ABSTRACT

The computation and construction of curved beams along freeform skins pose many challenges. We show how to use surfaces of constant mean curvature (CMC) to compute beam networks with beneficial properties, both aesthetically and from a fabrication perspective. To explore variations of such networks we introduce a new discretization of CMC surfaces as quadrilateral meshes with spherical vertex stars and right node angles. The computed non-CMC surface variations can be seen as a path in design space – exploring possible solutions in a neighborhood, or represent an actual erection sequence exploiting elastic material behavior.

CCS CONCEPTS

• **Mathematics of computing → Mathematical optimization**;
• **Computing methodologies → Shape analysis**;

KEYWORDS

CMC surfaces, support structures, elastic deformation

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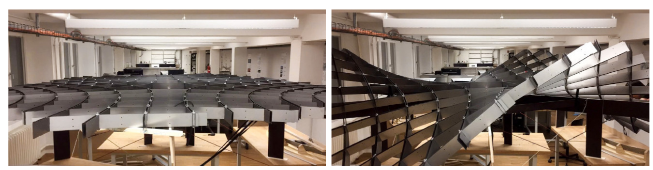

Figure 2: Elastic deformation of a lattice into a beam network following asymptotic directions on a minimal surface.

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1 INTRODUCTION

Gridshells are highly efficient structures, because they carry loads through their curved shape with very little material. For example, the INSIDE\OUT pavilion, unveiled at TU Munich in November 2017, was built from steel lamella with straight unrolling. The initially planar lamella grillage can be deformed elastically (see Fig. 2), allowing an erection process without scaffolding. Geometrically, this is a family of grids with *planar vertex stars* that assumes the shape of a *minimal surface* when node angles reach 90 degrees. We generalize this idea to quadrilateral meshes with *spherical vertex stars*, allowing circular unrollings of strips.

2 APPROACH

We model curved support structures as networks of developable strips attached *orthogonally* to a design surface. Such a network is an idealized representation of the center planes of beams. To cover a design surface with beams of straight or circular development, individual curves of the network need to follow curves of constant normal curvature. In addition to this geometric property one also has to satisfy aesthetic requirements related to the layout of curves, for instance, curves close to 90 degree intersection angles, as-square-as-possible faces, or equal edge lengths throughout the network. On a generic design surface a network satisfying all those properties may not exist. We propose a method that restricts the designer to a subset of the space of admissible shapes. CMC surfaces serve as starting shapes as they can be covered by a network of curves with constant normal curvature, featuring close to right angle intersections and almost square faces. Such networks are close to meshes with *spherical vertex stars* and generalize the notion of meshes with planar vertex stars appearing in the study of asymptotic nets [Bobenko and Suris 2008]. More precisely, we discretize CMC surfaces as quad meshes such that: (i) a vertex and its four neighbors lie on a common sphere, (ii) all such sphere radii are equal and (iii) edge polylines intersect at right angles. Preserving the spherical vertex star property and the constant radius during deformation allows us to explore the shape of nearby support structures (which no longer represent CMC surfaces):

• Starting from a CMC surface, we compute an isothermic mesh M on top of it. This yields a highly aesthetic cell layout, plus, by following the diagonals of M, we can extract a support structure S along curves of constant normal curvature, allowing for beams with straight or circular unrolling.

• By construction, the quad-dominant mesh S is close to a mesh with spherical vertex stars. We optimize S to satisfy the spherical vertex star condition in the least squares sense. Optimization will also ensure that all vertex spheres have the same radius.

• In order to study elastic shape variations of S we preserve (i) the spherical vertex star condition, (ii) the radii of spheres and (iii) edge lengths of S during shape editing. Optimization is done using the method presented in [Tang et al. 2014].

3 RESULTS AND CONCLUSION

We used an unduloid surface to initialize the network shown in Fig. 1(a). The deformation was achieved by moving a ring of vertices at half the height of the structure inwards. Our method can also be used to identify an elastic erection sequence similar to the process illustrated in Fig. 2. An example is shown in Fig. 3.

The presented approach complements work on tangent strips by [Pottmann et al. 2008] and presents further insights to research initiated by [Tang et al. 2016]. Our work is purely based on geometric considerations and needs to undergo mechanical and structural analysis in order to be considered an engineering solution to the computation of curved support structures. Incorporation of mechanical properties into our optimization, similar to [Kilian et al. 2017], is an interesting direction for future work.

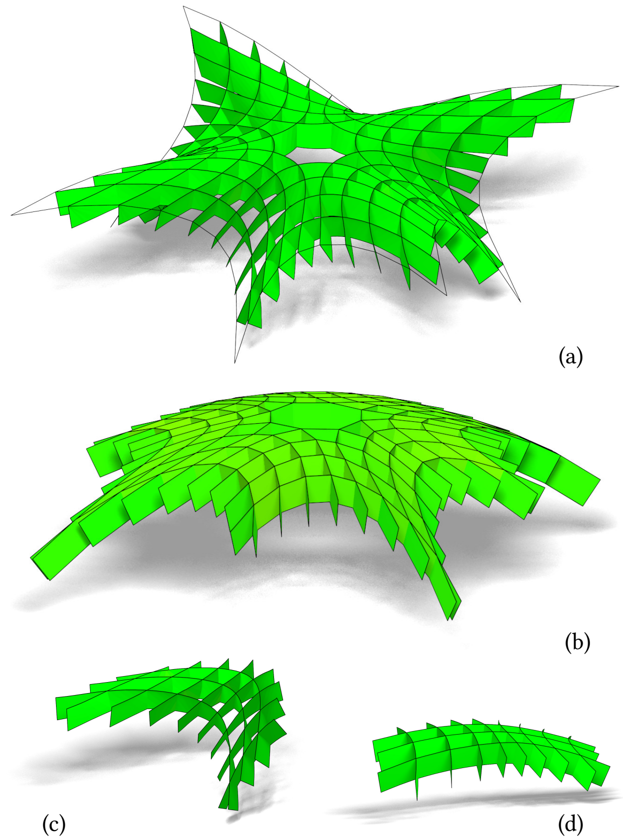

Figure 3: (a) Mesh with spherical vertex stars and associated support structure. (b) Global 'unrolling' onto a common sphere. (c) Lower left part of the structure and (d) its 'unrolling'. The common sphere can be thought of as scaffolding during construction before the elastic erection process starts.

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