Improving Incompressible SPH Simulation Efficiency by Integrating Density-Invariant and Divergence-Free Conditions

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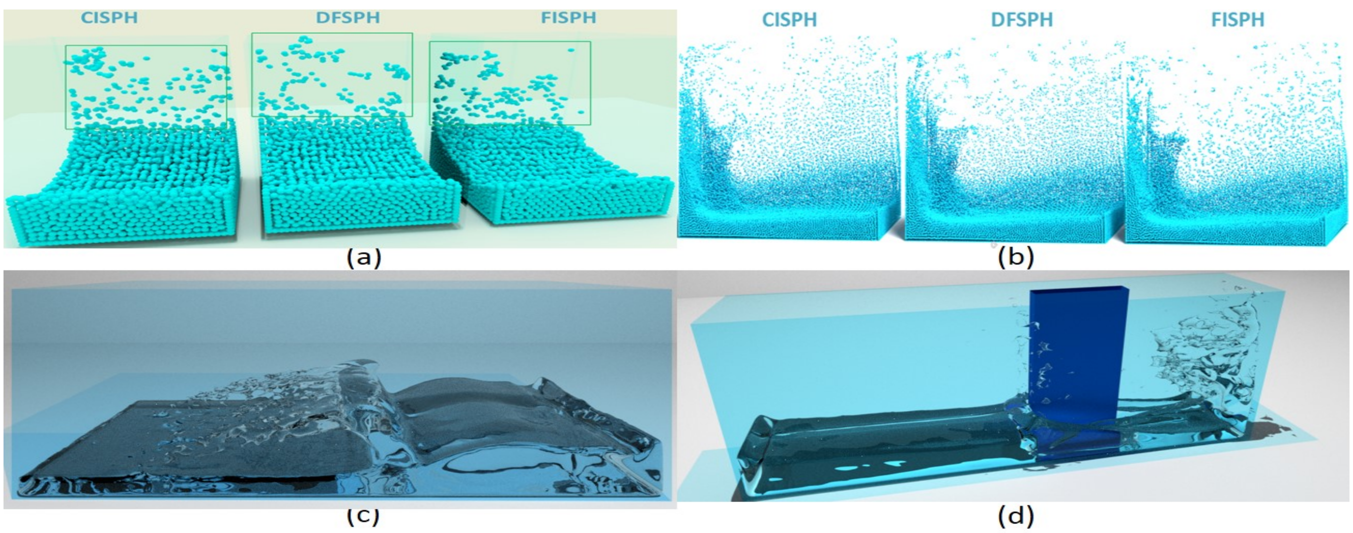

Figure 1: (a) and (b) show the particle distribution of our CISPH is more regular and homogeneous than that of the DFSPH and the FISPH with the 4.8K and 70.4K particles, respectively. (c) Breaking dam model with 576K particles. (d) Breaking dam model (192K fluid particles) with a cuboid obstacle.

ABSTRACT

Our method shortens the time of fluid simulation by coupling the two conditions of density-invariant and divergence-free, and achieves the same simulation effect compared with other methods. Further, we regard the displacement of particles as the only basic variable of the continuity equation, which improves the stability of the fluid to a certain extent.

CCS CONCEPTS

• **Computing methodologies** → Animation; Physical simulation;

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KEYWORDS

SPH, time-consuming, density-invariant, divergence-free

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1 INTRODUCTION

In the field of graphics, to maintain the incompressibility of volume, most of the Smoothed Particle Hydrodynamics (SPH) methods based on the continuity equation of incompressible fluids, which are proven to be effective, focus on the correction of density error and the divergence-free velocity field. However, these two conditions are considered separately in the existing methods, which leads to a time consuming process when the algorithm is used to deal with the continuity equation. In addition, the two conditions that are treated separately can also cause the continuity equation to be

Table 1: The lowest total computation times are marked bold.

Particles number	Density-Invariant(DI) and Divergence-Free (DF)					
	DFSPH		FISPH		CISPH (Ours)	
	time/iter.[ms]	time/frame[ms]	time/iter.[ms]	time/frame[ms]	time/iter.[ms]	time/frame[ms]
4,800	74.11 (DI+DF)	149.073	63.243 (DI+DF)	127.286	**31.322**	**62.344**
7,0400	2153.251 (DI+DF)	4300.530	1894.214(DI+DF)	3803.016	**910.238**	**1811.432**

inconsistent in the parameters and variables, which results in the instability of the fluid.

In this paper, the density-invariant and the divergence-free will be coupled to compute for improving incompressible SPH. We take the particle's position as the only basic variable in the continuity equation As show in Fig. 1(a)(b), the particle distribution of our coupling incompressible SPH (CISPH) is more regular and homogeneous than those of DFSPH [Bender and Dan 2015] and FISPH [Kang and Sagong 2015]. More importantly, the total computation in the time consumption per iteration is reduced nearly halves than DFSPH and FISPH in Fig. 1.

2 OUR APPROACH

To avoid the two conditions of the density-invariant and divergence-free applied separately, we consider the two conditions together in continuity equation to guarantee the stability of fluid and save time. By formulating and solving a set of positions constraints based on Position Based Dynamics framework (PBD) [Müller et al. 2007], we correct the continuity equation error after the time integration.

The equation of continuity solver which satisfies the divergence-free condition and the density-invariant condition. In the beginning, we find the adjacent particles of each particle i mentioned. The ρ_i depends on the distance between particles, the support of the kernel function and particle mass. Since, the constant density and divergence-free are satisfied and the particle positions are modified by the continuity equation. The method recomputes particle positions at every time step and recalculates continuity equation errors for each iteration.

CISPH based on PBD predict the continuity equation error using the time rate of change of the density. The formulation is obtained by directly discretizing the continuity equation $\frac{D\rho_i}{Dt} = -\rho_i \nabla \cdot \mathbf{v_i}$. The density ρ_i at a point in time t is computed from $t - \Delta t$ using the the first order Taylor approximation, which yields

$$\begin{aligned}\rho_i(t) &= \rho_i(t-\Delta t) + \Delta t \frac{D\rho_i}{Dt} \\ &= \rho_i(t-\Delta t) + \Delta t \sum_j m_j \mathbf{v_{ij}}(t) \cdot \nabla W_{ij}(t) \\ &= \rho_i(t-\Delta t) + \sum_j m_j \Delta \mathbf{x_{ij}}(t) \cdot \nabla W_{ij}(t).\end{aligned} \tag{1}$$

Using the $\Delta \mathbf{x_{ij}}(t) = \Delta t(\mathbf{v_i}(t) - \mathbf{v_j}(t)) = \Delta \mathbf{x_i}(t+\Delta t) - \Delta \mathbf{x_j}(t+\Delta t)$ and $\rho_i = \rho_0$ we get

$$\begin{aligned}\rho_i(t) = \rho_0 + \sum_j m_j[(\mathbf{x_i}(t+\Delta t) - \mathbf{x_j}(t+\Delta t)) \\ +(\mathbf{x_j}(t) - \mathbf{x_i}(t))] \cdot \nabla W_{ij}(t),\end{aligned} \tag{2}$$

The constraint C_i for enforcing the divergence-free condition and density-invariant condition can be computed as

$$\begin{aligned}C_i = 0 \Leftrightarrow \rho_i = \rho_0, |\sum_j m_j[\mathbf{x_i}(t+\Delta t) - \mathbf{x_j}(t+\Delta t) \\ +\mathbf{x_j}(t) - \mathbf{x_i}(t)] \cdot \nabla W_{ij}(t)| = 0.\end{aligned} \tag{3}$$

$C_i = 0$ means that CISPH satisfies both the divergence-free condition and the density-invariant condition.

Inspired by the PBD, we will show how to find our density and divergence correction that makes the constraint zero:

$$C(\mathbf{x} + \Delta \mathbf{x}) = 0. \tag{4}$$

By applying Taylor expansion, this equation can be approximated

$$C(\mathbf{x} + \Delta \mathbf{x}) \approx C(\mathbf{x}) + \lambda(\nabla C)^T \nabla C + \varepsilon\lambda = 0. \tag{5}$$

Restricting $\Delta \mathbf{x}$ to be in the direction of $\nabla_{\mathbf{x}} C$ means choosing a scalar λ such that

$$\Delta \mathbf{x} = \lambda \nabla C. \tag{6}$$

Substituting 6 into Eqn. 5 yields the final formula for Δx_i

$$\Delta \mathbf{x}_i = -\frac{C_i(\mathbf{x})}{|\nabla_{\mathbf{x}_i} C_i(\mathbf{x})|^2 + \varepsilon} \nabla C_i \tag{7}$$

If $\rho_i >= \rho_0$ and $\sum_j m_j[\mathbf{x_i}(t+\Delta t) - \mathbf{x_j}(t+\Delta t) + \mathbf{x_j}(t) - \mathbf{x_i}(t)] \cdot \nabla W_{ij}(t) >= 0$, the derivative of the constraint C_i can given below.

$$\nabla_{\mathbf{x}_k}(t) C_i = \begin{cases} \begin{aligned} &\sum_j m_j \nabla_{\mathbf{x}_k}(t) W_{ij}(t) + \\ &\sum_j m_j \nabla_{\mathbf{x}_k}\{[\mathbf{x_i}(t+\Delta t)) - (\mathbf{x_j}(t+\Delta t) \\ &+\mathbf{x_j}(t) - \mathbf{x_i}(t)] \cdot \nabla W_{ij}(t)\} \end{aligned} & k = i, \\ \begin{aligned} &m_j \nabla_{\mathbf{x}_k}(t) W_{ij}(t) + \\ &m_j \nabla_{\mathbf{x}_k}\{[\mathbf{x_i}(t+\Delta t) - \mathbf{x_j}(t+\Delta t) \\ &+\mathbf{x_j}(t) - \mathbf{x_i}(t)] \cdot \nabla W_{ij}(t)\} \end{aligned} & k = j. \end{cases}$$

Similarly, the other three cases can be computed in a similar manner.

So, the constraint condition $C_i(\mathbf{x}_1(t), \mathbf{x}_2(t), ..., \mathbf{x}_n(t))$ at time t taking the particle's position as the basic variable and other variables are replaced by constants.

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