Fractal Anatomy: Imaging Internal and Ambient Structures

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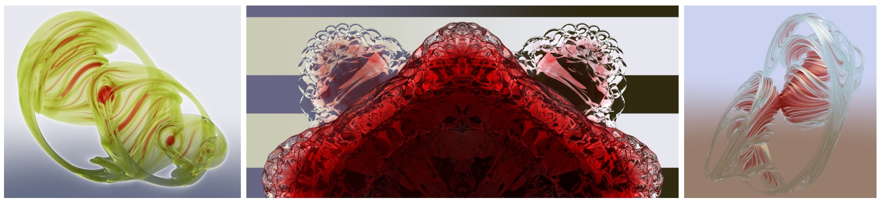

Figure 1: Glass Julia sets (left, right) and glass Mandelbulb (center) with orbit trapping.

ABSTRACT

Fractal shapes reflect the behavior of complex natural systems, but can be generated by simple mathematical equations. Images of 3D fractals almost exclusively depict opaque surfaces, and use reflected light and shadows to simulate a physical realization of these virtual objects. But rich inner detail can be revealed by reinterpreting the fractal as a volume and considering material transparency, light absorption and refraction. This work explores the range of images made possible by employing volume rendering techniques inspired by medical image visualization.

CCS CONCEPTS

• **Applied computing** → **Media arts**; • **Computing methodologies** → *Ray tracing*; • **Human-centered computing** → *Scientific visualization*;

KEYWORDS

generative art, computational art, fractal

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1 INTRODUCTION

Benoit Mandelbrot was the father of fractal geometry, and, in fact, coined the term 'fractal'. In 1979, the first crude dot-matrix printouts of the Mandelbrot set took hours to compute [Mandelbrot 1983]. At the 1989 SIGGRAPH Art Show, Mandelbrot proposed that fractal geometry had initiated a new category of art: "art for the sake of mathematics" - a field straddling the intersections of creativity, invention and discovery [Mandelbrot 1989].

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In the past decade, the fractal art community has created / invented / discovered new systems of equations which generate 3D surfaces which have seemingly infinite detail. Images of the Mandelbulb and Mandelbox fractals, named in honor of Mandelbrot, abound in the fractal art community. The images, however, almost always represent a solid opaque structure. This representation occludes many details of the fractal shapes, such as back-facing geometry and interior features. In this work, we explore the use of transparent materials to represent fractals, using volume rendering techniques to explore the anatomy of fractals as revealed by medical image visualization techniques, and by raycasting the surfaces while simulating light absorption and refraction as if the shapes were sculpted from glass.

2 BACKGROUND

The mathematical field of fractal geometry has been used to describe the complex appearance of natural systems, such as branching plants and turbulent fluids. Spiritual imagery, such as Hindu mandalas, Celtic knots, and Islamic geometric patterns also have fractal designs. Fractal art is often considered a subset of algorithmic art, but the concepts of fractal geometry have influenced artists working in traditional media, as highlighted by Susan Condé [Condé 2001] in 2001. Paintings by Edward Berko and Carlos Ginzburg [Ginzburg 2001] embody the fractal property of self-similarity across multiple scales. Susan Derges's photographs feature turbulent water and the fractal branching structures of trees and leaves.

The 2D Mandelbrot set [Mandelbrot 1983] and Julia sets are computed using simple recursive formulas involving complex numbers. The actual 'sets' are the collections of points in the complex plane which do not escape to infinity during iteration. Early black and white images of the Mandelbrot set reflected only whether a point was in the set or not. Later, color images were defined by computational parameters, such as mapping the iteration counter to color. The coloring approach known as "orbit trapping" [Carlson 1999] is based on the minimum distance between the iterated sequence of points (the so-called "orbit") and a preselected geometric shape (e.g. a circle or line). John Hart et al. [Hart et al. 1989] developed a method for raytracing 3D fractals by estimating the distance to the surface from an arbitrary point in space, which permits empty

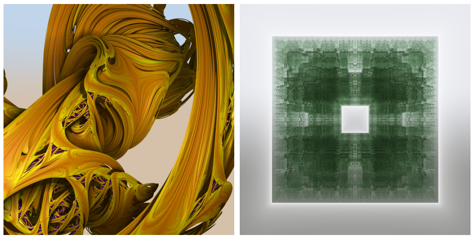

Figure 2: Detail of the quaternion Julia set colored using orbit trapping (left), glass Menger sponge (right)

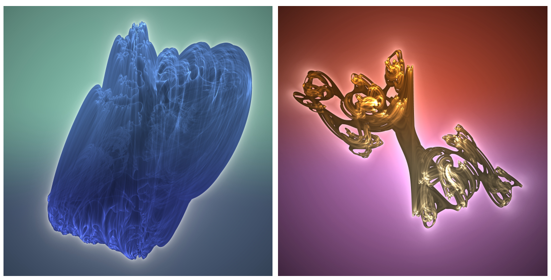

Figure 3: Volume rendered Julia sets

space to be efficiently skipped. The computer graphics demo scene [Scheib et al. 2002] has adopted this technique to great effect, creating real-time surface renderings of many fractals. By contrast, many volume rendering techniques for medical image visualization use a fixed ray step size, but allow rays to traverse through the dataset. We have combined aspects of both techniques to create a new body of work in which fractals are reinterpreted as transparent materials to reveal multiple layers and previously hidden structures.

3 METHODS

Unlike a complete raytracing solution, the raymarching technique we use does not generate numerous additional rays to find a global illumination solution. We begin from a single point for each pixel on the screen, and iteratively step along each ray by the estimated distance to the fractal. The color of each pixel in the image can be influenced by many factors, including light absorption within the interior of the fractal, partial reflection at the surface, and refraction as the ray intersects the fractal. Tracing rays through the volume, rather than halting at the surface, reveals the depth complexity of the fractal form, and allows the shape to bend light rays as they pass through the surface. This is at the expense of additional computational time. In the past decade, the generative art and fractal hobbyist communities have described many new 3D fractals. The Mandelbulb [White 2012] is computed by recursively scaling and rotating 3D points. The Mandelbox [Lowe 2010] is generated by repeatedly reflecting points about boxes and spheres.

4 RESULTS

In this section, we present works created using the previously described raycasting techniques, and describe the custom shader code variations designed for each image.

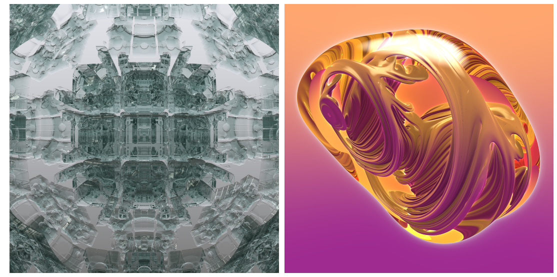

Figure 4: Mandelbox detail (left) and Julia set embedded in a transparent offset surface (right).

Figure (3) was created using a lighting model that attenuates due to object thickness, but also amplifies light when rays graze a surface. This results in emphasized internal boundaries highlighting the shape of a 2D Julia set.

Figure (4) was created by incorporating refraction into the raycasting process. When the viewing ray intersects the fractal, its direction is perturbed based on simulated index of refraction. Simple background patterns, like gradients or stripes, are distorted by the bending light rays. The transparent surface allows internal orbit trapping to be seen in Figure (3 left). Figure (3 right) was created by embedding an opaque fractal surface inside a refractive shell offset from the original surface of the fractal. The resulting images show warped views of the fractal and allow the ambient space surrounding the fractal to be visualized. Even though this ambient space represents points that escape after multiple iterations of the function, orbit trapping can still produce patterns here.

5 CONCLUSIONS

In this work, we explored the volumetric space inhabited by fractals by forming images based on light transmission, absorption, refraction and reflection. Our rendering methods were influenced by the fractal art community, the real-time graphics demo scene, medical image visualization techniques and glass artists, such as Dale Chihuly. In future work, we plan to develop new visualization and deformation techniques to probe the structure of fractals and other complex forms.

REFERENCES

Paul W Carlson. 1999. Two artistic orbit trap rendering methods for Newton M-set fractals. *Computers & Graphics* 23, 6 (1999), 925–931.

Susan Condé. 2001. The fractal artist. *Leonardo* 34, 1 (2001), 3–10.

Carlos Ginzburg. 2001. The Neuronal Network of Social Culture, Homo Fractalus. *Leonardo* 34, 1 (2001), 7–7.

John C Hart, Daniel J Sandin, and Louis H Kauffman. 1989. Ray tracing deterministic 3-D fractals. In *ACM SIGGRAPH Computer Graphics*, Vol. 23. ACM, 289–296.

Tom Lowe. 2010. What Is a Mandelbox? https://sites.google.com/site/mandelbox/what-is-a-mandelbox.

Benoit B Mandelbrot. 1983. *The fractal geometry of nature.* Vol. 173. WH freeman New York.

Benoit B Mandelbrot. 1989. Fractals and an art for the sake of science. *Leonardo* 22, 5 (1989), 21–24.

Vincent Scheib, Theo Engell-Nielsen, Saku Lehtinen, Eric Haines, and Phil Taylor. 2002. The demo scene. In *ACM SIGGRAPH 2002 conference abstracts and applications.* ACM, 96–97.

Daniel White. 2012. The unravelling of the real 3D Mandelbulb. http://www.skytopia.com/project/fractal/mandelbulb.html.