

Artifacts of Research: On Singularities

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Figure 1: Recreated series of Rodenberg models with lines of ruling c. 2008. ©Chertok 2008.

1. Introduction

Relatively recent developments in computer visualization have, among other things, focused increasing interest on a classical mathematical model collection originally made by hand in plaster in the 1860's. Much like the effort to classify the animal kingdom this was an effort to catalog the mathematical universe.

I welcome the opportunity to exhibit a full-size recreation of the largest series in this collection – the series designed to represent types of singularities possible on a cubic surface. These models were created by Carl Rodenberg for his thesis and produced under the direction of Felix Klein (1849 – 1925) the founder of modern topology. They are recreated here as plaster-based Rapid Prototyping (RP) models. Additionally presented are unique new models of the Clebsch Diagonal Cubic, a so-called "smooth" surface and the "god-head" of the series from which the series of singularities originate.

I would also like to direct your attention to a video of the Clebsch Diagonal Cubic which simultaneously demonstrates the complex geometric configuration of this famous surface and gives some insight into the Computer-Aided Design (CAD) origins of the work.

I am currently at work on a comprehensive, illustrated summary of the typology and mathematics behind these forms to accompany the exhibition.

2. Exposition

Beyond their sublime beauty, the new models address a surprisingly large number of current digital technology issues involving computational geometry, discrete differential geometry, algebraic geometry as well as their relation to nurbs, mesh, and reverse engineering. In this vein, three observations regarding technique are germane. First, the fact that the surfaces had to be modeled as mesh in mathematical modeling software (the only way to generate these zero sum implicit equations of the surfaces) while the straight lines were generated as nurbs lines in CAD software (from their parametric equations) resulted in the lines proving inherently more malleable and easy to manipulate as a consequence of their nurbs nature (i.e. mesh are sometimes hard to control). Second, the relative precision of the parametrically derived lines - when literally placed against the complicated curving

mesh surfaces generated by the Marching Cube Algorithm of the software - allowed for a check of the algorithm, which would have been difficult to discern otherwise. Third, the ability to "fly-through" using a 3D mouse provided unequalled power for querying both the model and the relationships inherent in it. Thus, the use of CAD to work with these models provided obvious visualization and interactive benefits as compared to conventional mathematical modeling software.

Similarly, the ability to work with actual models provides benefits. Even with a solid plaster-based RP model of the Clebsch Diagonal Cubic in your hand, it is still quite tricky to see, for instance, that the 27 lines on its surface are in fact straight, let alone to see their relationship amongst each other (the hyperboloid at the waist of this model creates an appearance of curvature). Utilizing newer RP model technologies can provide unique feedback in these and other respects.

On a computational note, I am currently working on a "script" that would generate all possible 36 combinations for the numbering of the lines on this surface based upon the actual CAD configuration and in conjunction with the historical documentation of the geometric relationships of these lines. I believe this could be used to check the numbering of the original Clebsch models' lines which were numbered in various ways in the 1800's.

3. Conclusion

The research represents an empirical exercise in design and critical thought and I am conducting parallel research on both the large scale fabrication of so-called "free-form" structures and in the emerging field of Architectural Geometry. I detail directions for future research in the more technically oriented Supporting Document.

Inevitably returning to my black and white photographs of the original models cause me to reflect upon the relationship of nature, beauty and making things. Perhaps as Robin Evans put it in another context, "by using more geometry they appear to have less".

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May 2008
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