Delaunay Lofts: A New Class of Space-Filling Shapes

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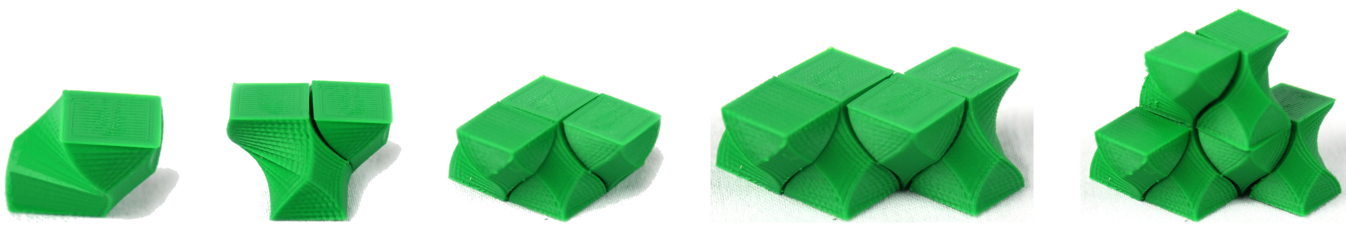

(a) One tile. **(b) Two tiles.** **(c) Three tiles.** **(d) Four tiles.** **(e) Seven tiles.**

Figure 1: An example of a single Delaunay Loft tile that can fill 3-space. This tile is created as an interpolation of two layers of tilings, namely (1) a square tiling; and; (2) another square tiling, which is a translation of the first square tiling. The interpolating control curves are straight lines.

ABSTRACT

We have developed an approach to construct and design a new class of space-filling shapes, which we call *Delaunay Lofts*. Our approach is based on interpolation of a stack of planar tiles whose dual tilings are Delaunay diagrams. We construct control curves that interpolate Delaunay vertices. Voronoi decomposition of the volume using these control curves as Voronoi sites gives us lofted interpolation of original polygons in planar tiles. This, combined with the use of wallpaper symmetries allows for the design of space-filling shapes in 3-space. In the poster exhibition, we will also demonstrate 3D printed examples of the new class of shapes (See Figures 1 and 3).

CCS CONCEPTS

• **Computing methodologies → 3D VR Interface**;

KEYWORDS

Space Filling Shapes, Voronoi Decomposition, Delaunay Diagrams, Lofting

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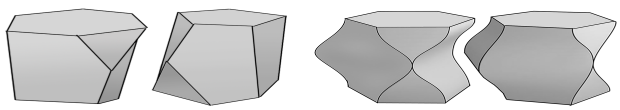
1 INTRODUCTION

In this work, we briefly present our new approach to design a large class of space filling shapes. A space-filling shape is a cellular structure whose replicas together can fill all of space watertight, i.e. without having any voids between them, or equivalently, it is a cellular structure that can be used to generate a tessellation of space. Although 2D space filling shapes are well-understood, 3D space filling shapes still pose many open questions. We now know that there are only eight space-filling convex polyhedra and only five of them have regular faces, namely the triangular prism, hexagonal prism, cube, truncated octahedron, and Johnson solid gyrobifastigium . It is also interesting that five of these eight space filling shapes are "primary" parallelohedra, namely cube, hexagonal prism, rhombic dodecahedron, elongated dodecahedron, and truncated octahedron. Our work significantly extend the family of space filling shapes using curved faces and edges.

2 RELATED WORK

Our approach, which can be considered as a generalization of parallelohedra, is inspired by a recent discovery by Gómez-Gálvez et al. who observed a simple polyhedral form, which they call *"scutoids"*, commonly exist in epithelial cells in the formation of thin skin

layers [Gómez-Gálvez et al. 2018]. The literature on this discovery shows the occurrence of scutoids and provides some statistical information about when and how they form (See 2). The scutoidal shapes can be considered as interpolation of 2D tiling patterns, that usually consists of hexagons and pentagons that appear on many natural structures. Interpolation is obtained by edge-collapse and vertex-split operations.



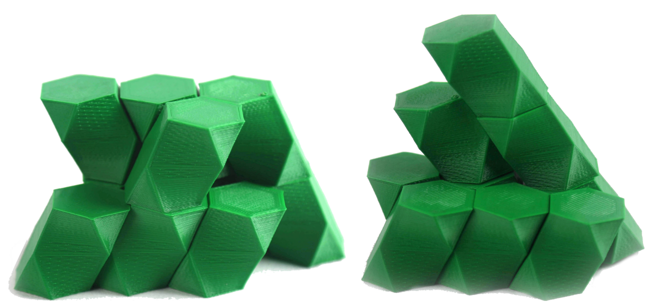
(a) Two views of a scutoid. **(b) Two views of a Delaunay loft.**

Figure 2: The comparison of scutoids with our Delaunay Lofts. The original scutoids usually depicted using straight edges as shown in this visual representation. Delaunay Lofts, on the other hand, (1) have curved edges and (2) can fill space.

The Figure 2a demonstrates a usual depiction of the originally discovered scutoid structures obtained by edge-collapse or vertex-split operations between pentagons and hexagonal faces. This view results in non-planar pentagons or hexagons with straight boundaries as shown in the Figure 2a, but it does not provide any well-defined process to fill the inside of these non-planar faces. Our approach to obtaining scutoid-like structures is to use 3D Voronoi decomposition using a set of curves as Voronoi sites. If this set of curves is closed under symmetry operations, the resulted Voronoi shapes are guaranteed to be space filling. It is interesting to note that this approach is also in sync with Delaunay's original intention for the use of Delaunay diagrams. Delaunay was, in fact, the first to use symmetry operations on points (instead of curves) and Voronoi diagrams to produce space-filling polyhedra, which he called Stereohedra. Our approach can be viewed as an extension of his idea to curves. We, therefore, called our approach Delaunay Lofting.

3 METHODOLOGY AND IMPLEMENTATION

When using points, construction of 3D Voronoi decomposition is relatively simple since distances to points guarantee to produce planar faces. On the other hand, when we use curves or even straight lines Voronoi decomposition can produce curved faces, which, in fact, makes this method interesting. However, having curved faces significantly complicates the algorithms to construct 3D Voronoi decomposition in high resolution. We, therefore, choose to deal with a subset of this general problem. We decompose thin rectangular structures that consist of a discrete set of z-constant planar layers. We also choose the control curves in the form of $(x_i = f_{i,x}(z), y_i = f_{i,y}(z))$, where $i = 0, 1, \ldots, n$. This constraint guarantees that each curve intersects with each layer only once. We also use a specific distance function to further simplify the process into a set of 2D Voronoi decomposition. Based on these simplifications, the general process consists of the following steps: (1) Discretize the rectangular prism with N number of constant z planes, which we call layers. (2) Design M number of curves inside of the rectangular domain. (3) Find the intersection of curves with intermediate layers. (4) For each layer, compute its Voronoi partitioning by using intersection points with that particular layer as Voronoi sites. Since space is bounded, the boundaries of the prism become part of Voronoi polygons. For regular domain, we compute Voronoi decomposition in a 2-toroidal domain.. (5) Offset each Voronoi polygon the same amount using Minkowski difference. (6) Treat each vertex as a single manifold and insert edges between consecutive vertices. This process turns each original face into a 2-sided face [Srinivasan et al. 2002], which is actually a 2-manifold. (7) Insert edges between closest vertices in consecutive layers based on face normal. This process automatically creates the Delaunay Lofts and the resulting structures resemble scutoids with curved edges and faces. To produce space filling tiles, control curves must be closed under symmetry operations and each rectangle must be a regular domain, which topologically forms a 2-toroid. To easily produce control curves that are closed under symmetry operations, we interpolate 2D Delaunay diagrams with one of the 17 wallpaper symmetries. If the top and bottom tilings are rigid transformations of each other, this process guarantees to produce 3D space filling shapes. In the 3D printed examples shown in Figures 1, and 3, control curves are just straight lines that interpolate top and bottom Delaunay vertices. We also have examples that interpolate more than two tiles, one such example is shown in Figure 2b. The design space of space filling shapes that can be composed using this approach is unusually rich. This is due to the fact that the construction algorithm does not assume any specific shape of the control curves — as long as they intersect each slicing plane at a unique point thus maintaining the number of sites per slice.



(a) Ten tiles. **(b) Ten tiles.**

Figure 3: Another example of single space filling Delaunay Loft tile. This tile is created as an interpolation of three layers of tilings, namely (1) a regular hexagonal; (2) a square and; (3) another regular hexagonal tilings, which is a translation of the first hexagonal tiling.

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