Delaunay Lofts: A New Class of Space-Filling Shapes

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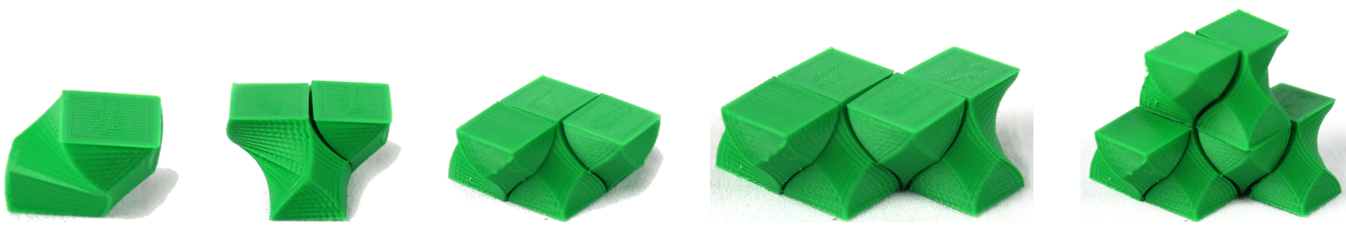

(a) One tile. **(b) Two tiles.** **(c) Three tiles.** **(d) Four tiles.** **(e) Seven tiles.**

Figure 1: An example of a single Delaunay Loft tile that can fill 3-space. This tile is created as an interpolation of two layers of tilings, namely (1) a square tiling; and; (2) another square tiling, which is a translation of the first square tiling. The interpolating control curves are straight lines.

ABSTRACT

We have developed an approach to construct and design a new class of space-filling shapes, which we call *Delaunay Lofts*. Our approach is based on interpolation of a stack of planar tiles whose dual tilings are Delaunay diagrams. We construct control curves that interpolate Delaunay vertices. Voronoi decomposition of the volume using these control curves as Voronoi sites gives us lofted interpolation of original polygons in planar tiles. This, combined with the use of wallpaper symmetries allows for the design of space-filling shapes in 3-space. In the poster exhibition, we will also demonstrate 3D printed examples of the new class of shapes (See Figures 1 and 3).

CCS CONCEPTS

• **Computing methodologies → 3D VR Interface**;

KEYWORDS

Space Filling Shapes, Voronoi Decomposition, Delaunay Diagrams, Lofting

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1 INTRODUCTION

In this work, we briefly present our new approach to design a large class of space filling shapes. A space-filling shape is a cellular structure whose replicas together can fill all of space watertight, i.e. without having any voids between them, or equivalently, it is a cellular structure that can be used to generate a tessellation of space. Although 2D space filling shapes are well-understood, 3D space filling shapes still pose many open questions. We now know that there are only eight space-filling convex polyhedra and only five of them have regular faces, namely the triangular prism, hexagonal prism, cube, truncated octahedron, and Johnson solid gyrobifastigium . It is also interesting that five of these eight space filling shapes are "primary" parallelohedra, namely cube, hexagonal prism, rhombic dodecahedron, elongated dodecahedron, and truncated octahedron. Our work significantly extend the family of space filling shapes using curved faces and edges.

2 RELATED WORK

Our approach, which can be considered as a generalization of parallelohedra, is inspired by a recent discovery by Gómez-Gálvez et al. who observed a simple polyhedral form, which they call *"scutoids"*, commonly exist in epithelial cells in the formation of thin skin