Contents

General Information on the Tutorial

Course Syllabus

Summary: This mixed-level tutorial will deal with visualizable representations of quaternion features, technology, folklore, and applications. The introduction will focus on visually understanding quaternions themselves. Starting from this basis, the tutorial will proceed to give visualizations of advanced quaternion dynamics and optimization problems.

Prerequisites: Participants should be comfortable with and have an appreciation for conventional mathematical methods of 3D computer graphics and geometry used in graphics transformations and rendering. The material will be of most interest to those wishing to deepen their intuitive understanding of quaternion-based animation, moving coordinate frames, and 3D curves and surfaces appearing in graphics and scientific visualization applications.

Objectives: Participants will learn the basic facts relating quaternions to ordinary 3D rotations, as well as methods for examining the properties of quaternion constructions using interactive visualization methods. A variety of applications, including quaternion splines and moving coordinate frames for curves and surfaces, will be examined in this context. Finally, a few facts about the deeper relationship between quaternions and Clifford algebras in higher dimensions will be presented.

Outline: This is a two-hour tutorial and the material will be arranged approximately as follows:

I. *(45 min)* **Introduction to Rotation Representations.** Develop formulas and techniques for seeing how 2D rotations, orientation frames, and their time evolution equations can be visualized and studied using ordinary complex variables. Develop the parallel relationship between 3D rotations and quaternions.

II. *(15 min)* **Visualization Techniques for Quaternions.** Visualizing static and moving quaternion frames as 4D geometric objects.

III. *(45 min)* **Applications of Quaternion Visualization.** Extend this intuition into the quaternion representation of 3D rotation splines and moving orientation frames for curves and surfaces.

IV. *(15 min)* **Clifford Algebras: the Bigger Picture.** Start to see how it all fits into Clifford algebras.