Iterated Function Systems and Recurrent Iterated Function Systems

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May 14, 1996

Chapter 1

ITERATED FUNCTION SYSTEMS

Excerpted from: J.C. Hart, “Computer Display of Linear Fractal Surfaces” Ph.D. Dissertation, EECS Dept., University of Illinois at Chicago, Sept. 1991.

The term “iterated function system” (abbreviated: IFS) was coined in [Barnsley & Demko, 1985] to describe a general framework of dynamics. However, most of the results about the IFS model were presented in [Hutchinson, 1981].

None of the theorems or proofs in this chapter are due to the author. However, the wording, layout and techniques used to make these theorems and their proofs coherent within a single chapter are new. Two examples: First, the proof of Theorem 1.4 is based entirely on the second half of the proof of the Blaschke selection theorem found in [Falconer, 1985]. Compare this short proof to the five part, two page proof of Theorem 2.7.1 in [Barnsley, 1988]. Second, the definition of “overlapping construction” from [Barnsley, 1988] only applies to connected sets and requires a difficult code space argument. Here, its definition is extended and uses only the simple terminology of metric spaces.

The term “iterated function system” is defined far from the beginning of the chapter. This placement makes it easier to explain why this definition may differ from definitions by other authors. Immediately following it is the proof that every IFS specifies a unique set, called its “attractor.” This theorem is titled the “Fundamental Theorem of Iterated Function Systems,” for it is the attractors that make iterated function systems interesting.

1.1 Lipschitz Functions, Contractions and Similtudes

A map is a function from one space into another. Most maps in this discussion take a metric space into itself. The category of map properties defined in this section describe the relationship of the distances between two points before and after a map is applied.

1.1.1 The Lipschitz Property

A map is “Lipschitz” when the ratio of the distances of the images of two points to that of the original two points is bounded by a constant. In other words, the images of points under a Lipschitz map can only get so far from each other, depending on how close they were before.