

## **Warping and Morphing of Graphical Objects**

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**N O T E S**

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# Warping and Morphing of Graphical Objects

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Course Notes – SIGGRAPH 1995

## **Abstract**

This course makes a comprehensive study of the problem of warping and metamorphosis of graphical objects. It presents a unified view of the metamorphosis problem involving drawings, surfaces, images, volume data, etc. A detailed discussion of the concept of a graphical object is introduced in the course. This is necessary to create a solid foundation preparing the basis for the introduction of a unified framework involving the problem of warping and morphing of graphical objects. We try to keep the mathematics simple so that the material will be understood by a large audience with different background. At the end of the course we will discuss the use of metamorphosis techniques in the entertainment industry.

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## Lecturers Biographies

**John Andrew Berton, Jr.** is a Computer Graphics Supervisor at Industrial Light and Magic in Marin County, California. His feature film credits include "Casper," "Terminator2: Judgment Day" and "Star Trek VI - The Undiscovered Country." Previous to joining ILM in 1990, Berton created computer animation and electronic music for both commercial and artistic productions, including work for Cranston/Csuri Productions, The Ohio Supercomputer Center, The Ohio State University, and mental images, GmbH. Berton holds degrees in Communication and Film, (B.A. 1977 Denison University) and in Art Education/Computer Graphics (M.A. 1984 The Ohio State University.)

**Bruno Costa** has a computer engineering and M.Sc. degree in computer science from PUC at Rio de Janeiro, Brazil. He is currently a Ph.D. student in computer science at SUNY Stony Brook. He has been involved with the Visgraf project at IMPA since its beginnings in 1990, where he has worked on realistic rendering, image and implicit objects transformations, warping and morphing in particular. He has co-authored a commercial morphing software, as a by-product of his Master's research. His current interests also include computer graphics aspects of interactive environments, and transformations of volumetric objects.

**Lucia Darsa** is currently working towards a Ph.D. degree at SUNY at Stony Brook. She graduated in Computer Engineering at Pontificia Universidade Católica do Rio de Janeiro, Brazil, obtaining her M.Sc. degree in Computer Science in 1994 from a joint program between PUC-RJ and IMPA. She has been a member of the VisGraf project at IMPA since 1990, having worked with implicit surfaces rendering and modeling, motion control hardware and compositing, and image processing. She has been involved with morphing research and development since 1992, being a co-author of Visionaire, a commercial warping and morphing software.

**Jonas Gomes** is an associate professor at the Institute of Pure and Applied Mathematics - IMPA, Rio de Janeiro. He took a Phd in Mathematics from IMPA in 1984. He worked as the R&D manager of the computer graphics group at Globo TV network from 1984 to 1988. In 1989 he started a computer graphics group at IMPA. This group has been involved with research and post-graduate studies in graphics, and has given a noticeable contribution to the development of computer graphics in Brazil. He has co-authored four books on graphics and image processing, and he has published research papers in computer graphics and image processing. In 1993 he organized the course "Modeling on Graphics" at SIGGRAPH. His current research interests include Mathematical Foundations of Computer Graphics, Modeling, Visualization and Image Processing.

**Luiz Velho** is an Associate Researcher at IMPA – Instituto de Matematica Pura e Aplicada. He received a BE in Industrial Design from ESDI – Universidade do Rio de Janeiro in 1979, a MS in Computer Graphics from the Massachusetts Institute of Technology, Media Laboratory in 1985, and a Ph.D. in Computer Science in 1994 from the University of Toronto. His experience in computer graphics spans the fields of modeling, rendering, imaging and animation. During 1982 he was a visiting researcher at the National Film Board of Canada. From 1985 to 1987 he was a Systems Engineer at the Fantastic Animation Machine in New York, where he developed the company's 3D visualization system. From 1987 to 1991 he was a Principal Engineer at Globo TV Network in Brazil, where he created special effects and visual simulation systems. In 1994 he was a visiting professor at

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**George Wolberg** is an Associate Professor of Computer Science at the City College of New York/CUNY. He received the B.S. and M.S. degrees in Electrical Engineering from Cooper Union in 1985, and the Ph.D. degree in Computer Science from Columbia University in 1990. He has worked at AT&T Bell Laboratories and IBM T.J. Watson Research Center during the summers of 1983/4 and 1985/9, respectively. His research at these labs centered on image restoration, image segmentation, graphics algorithms, and texture mapping. Prof. Wolberg is the recipient of a 1991 NSF Presidential Young Investigator Award. He is the author of "Digital Image Warping," (IEEE Computer Society Press, 1990), the first comprehensive monograph on warping and morphing. His research interests include image processing, computer graphics, and computer vision.

## Sessions

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Introduction	Jonas Gomes
Fundamentals	Luiz Velho
Graphical Objects	Jonas Gomes
Transformation of Graphical Objects	Bruno Costa
Specification of Transformations	Lucia Darsa
Computation of Transformations	George Wolberg
Applications in the Entertainment Industry	John Berton

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