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COURSE NOTES

C26

BLOSSOMING THE NEW
POLAR-FORM APPROACH
TO SPLINE CURVES AND
SURFACES

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Lecturers
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Course Abstract

Polar forms provide a new labeling scheme for spline curves and surfaces that allows for elegant formulations of otherwise complicated results, and has led to several new algorithms. This course provides an in-depth introduction to polar forms and presents state-of-the-art techniques for constructing spline curves and surfaces from this unified and coherent perspective. Topics include polar forms for polynomial curves, new labels for curves and surfaces, Bézier curves and B-splines, dual functionals, knot insertion, geometric continuity of curves, universal splines, interpolation vs approximation, quasi-interpolants, rational curves, NURBS, tensor-product surfaces, triangular Bézier surfaces, B-patches, and triangular B-spline surfaces.

Speaker Biographies

Phillip J. Barry is an Assistant Professor of Computer Science at the University of Minnesota. He has a B.S. and M.S. in mathematics from Idaho State University, a Ph.D. in mathematics from the University of Utah, and did postdoctoral work in the Computer Graphics Lab at the University of Waterloo. His current work at Minnesota is in the area of computer aided geometric design.

Ron Goldman is currently a Full Professor of Computer Science at Rice University. He received his B.S. in Mathematics from MIT in 1968, and his M.A. and Ph.D. in Mathematics from Johns Hopkins University in 1972 and 1973. His work includes research in computer aided geometric design, solid modeling, computer graphics, and splines. He has published over forty articles in journals, books, and conference proceedings, and is an associate editor of the journals *Computer-Aided Design* and *Computer Aided Geometric Design*. Before returning to academia, Goldman worked for ten years in industry at Manufacturing Data Systems, Inc., as a Senior Design Engineer at Ford Motor Company, and as a Principal Consultant at Control Data Corporation. He became an Associate Professor of Computer Science at the University of Waterloo in 1987, and joined Rice University in 1990.

Lyle Ramshaw majored in Mathematics at Harvard. He completed his Ph.D. work in Computer Science at Stanford, under Prof. Donald E. Knuth in 1979. In his thesis, he developed techniques for formalizing the analysis of algorithms, that is, for verifying the average-case running time of a program, as opposed to verifying its correctness. He worked for five years at the Palo Alto Research Center of the Xerox Corporation, then moved to the Systems Research Center of the Digital Equipment Corporation. Recently, he has been focusing on problems in computer-aided geometric design.

Hans-Peter Seidel is an assistant professor of computer science at the University of Waterloo, Canada. He received a Ph.D. in Mathematics and a Habilitation in Computer Science from Tübingen University in 1987 and 1989, and joined the University of Waterloo in 1989. His current research interests include computer graphics, computer-aided geometric design, freeform curves and surfaces, and geometric modeling. Seidel is a member of the editorial board of *Computer Graphics Forum* and co-editor of the book *Theory and Practice of Geometric Modeling*.

Schedule

Lecture 1**Title:** The Polar Forms of Polynomial Curves**Speaker:** Lyle Ramshaw, DEC Systems Research Center**Time:** 60 minutes**Notes:** 1 1 - 1 29**Lecture 2****Title:** Blossoming and B-Splines**Speaker:** Ronald N Goldman, Rice University**Time:** 45 minutes**Notes:** 2 1 - 2 24**Lecture 3****Title:** The Relation of Polar Forms to Other B-Spline Approaches**Speaker:** Phillip J Barry, University of Minnesota**Time:** 45 minutes**Notes:** 3 1 - 3 27**Lecture 4****Title:** Knot Insertion Algorithms**Speaker:** Ronald N Goldman, Rice University**Time:** 45 minutes**Notes:** 4 1 - 4 53**Lecture 5****Title:** Geometrically Continuous Spline Curves**Speaker:** Hans-Peter Seidel, University of Waterloo**Time:** 30 minutes**Notes:** 5 1 - 5 18**Lecture 6****Title:** The de Casteljau Approach to Quasi-Interpolants**Speaker:** Lyle Ramshaw, DEC Systems Research Center**Time:** 30 minutes**Notes:** 6 1 - 6 22**Lecture 7****Title:** Polar Forms and Rational B-Spline Curves**Speaker:** Phillip J Barry, University of Minnesota**Time:** 45 minutes**Notes:** 7 1 - 7 36**Lecture 8****Title:** Polar Forms and Triangular B-Spline Surfaces**Speaker:** Hans-Peter Seidel**Time:** 60 minutes**Notes:** 8 1 - 8 52

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