

**SIGGRAPH 1991**  
**18th International Conference**  
**On Computer Graphics and**  
**Interactive Techniques**

**Las Vegas Convention Center**  
**28 July - 2 August**

## COURSE NOTES

## C25

TOPICS IN THE  
CONSTRUCTION,  
MANIPULATION, AND  
ASSESSMENT OF SPLINE  
SURFACES

***Chair***

**Richard Bartels**  
**University of Waterloo**

***Lecturers***

**Tony DeRose**  
**University of Washington**  
**David Forsey**  
**University of British Columbia**  
**David Warn**  
**General Motors Research**  
**Laboratories**

**Topics in the  
Construction, Manipulation,  
and Assessment  
of Spline Surfaces**

## **Course Abstract**

**The course will review basics for geometry, splines, and linear algebra. Rational and nonrational splines will be compared. Main surfaces categories – patches, tensor products, and hierarchies – will be introduced. Generation by sweeping, extrusion, rotation, curve-net filling, and data fitting will be reviewed. Surface manipulation will focus on direct control of features. Hierarchical surfaces will be described with their manipulation for design and animation. The construction of patches of special form and their smooth joining to surfaces will be detailed. The course will conclude with surface quality – continuity and curvature distribution – and its assessment and visualization.**

## Speaker Biographies

**Richard Bartels** is a Professor in the Department of Computer Science at the University of Waterloo and a member of Waterloo's Computer Graphics Laboratory. He received his Ph D from Stanford in 1968 and has been on the faculty of the University of Texas at Austin and The Johns Hopkins University before coming to Waterloo. His research interests are in the Computer Aided Geometric Design of surfaces. He is co-author, with John Beatty and Brian Barsky, of the book *An Introduction to Splines for Use in Computer Graphics and Geometric Modeling*.

**Tony D. DeRose** is currently an associate professor of computer science at the University of Washington. He received his B S in physics from the University of California, Davis in 1981 and his Ph D in computer science from the University of California, Berkeley in 1985. He received a Presidential Young Investigator award from the National Science Foundation in 1989. His research interests include the mathematical aspects of computer graphics and geometric modeling, and high level methods of shape representation. DeRose is an associate editor for ACM Transactions on Graphics, and is a member of SIGGRAPH, SIAM, and the IEEE Computer Society.

**David Forsey** is an Assistant Professor in the Department of Computer Science at the University of British Columbia and a director of Imager, UBC's Computer Graphics Laboratory. He received his Ph D from Waterloo in 1990 and has a BSc in Zoology from the University of Guelph. His research interests are in free-form surface design, user interfaces, and animation.

**David Warn** is a Senior Staff Research Scientist in the Computer Science Department at the General Motors Research Laboratories. He holds a B S in Mathematics from Carnegie-Mellon University and joined GM in 1968 after receiving a M S in Computer Science from Purdue University. His current research interests are in interactive surface design and evaluation, computer graphics, interactive techniques, and databases for computer aided design.

## Schedule

**3/4 hour, Tony DeRose (Pages 1-0 to 1-60 and following Appendix)**

The course will begin with a review of the elementary concepts of geometry, specifically affine geometry and geometric frames of reference

**1/2 hour, Richard Bartels (Pages 2-0 to 2-24)**

The basics of splines will be reviewed using the multiaffine polar form (blossom) The elements of rational splines will be introduced along with their connection to conic sections

**1/2 hour, Richard Bartels (Pages 3-0 to 3-12)**

Basic computational linear algebra will be covered linear equations, least squares, constrained least squares, and appropriate matrix transformations

**1/2 hour Richard Bartels (Pages 4-0 to 4-16)**

The direct manipulation of position, tangent, and curvature will be introduced as an example of applying the above material

**1 hour, David Forsey (Pages 5-0 to 5-56)**

*Editing is aided by the ability to add features at any level of detail in any order Small features should respond reasonably to modification of large, surrounding features* The hierarchical spline surface will be introduced and described, detailing the definition and use of overlays, offsets, and offset mapping functions Examples of non-rational and rational hierarchical B-splines will be shown using examples in applications from animation and human figure modeling

**1 hour, Richard Bartels (Pages 6-0 to 6-34 and Pages 7-0 to 7-14)**

*It is convenient to generate basic surface templates by intuitive operations There is frequently a need to model existing surfaces* Standard generation techniques will be covered sweeping, revolution, and extrusion The fitting of curve nets to data and the filling nets with patches will follow This section will end with the fitting of gridded data by tensor-product surfaces and the adaptive refinement of this fitting process to produce a hierarchical surface

**3/4 hour, Tony DeRose (Pages 8-0 to 8-9 and following Appendix)**

*For complicated geometry, four-sided patches are not enough* This section will cover generalized B-splines and multi-sided patches Surface construction methods that generalize bi-quadratic and bi-cubic tensor product B-splines will be introduced As these methods are based on a class of multi-sided patches known as S-patches, the section will begin with a survey of basic S-patch theory Algorithms for smoothly filling curve networks using S-patches will be emphasized

**1 hour, David Warn (Pages 9-0 to 9-14 and Pages 10-0 to 10-9)**

*In an industrial design setting, there is a practical need for surface quality* This section will present indicators of surface quality and methods for their visualization The section will include various surface evaluation techniques including shaded images, color coded surface curvature displays, line curvature plots, surface normals, and section cuts The discussion will focus on identifying problem areas and verification of quality in mathematical surfaces with some practical suggestions for methods of correction

# Contents

<b>Coordinate-Free Geometric Programming</b>	1-0
Introduction and Motivation	1-1
The Search for a Geometric Algebra	1-4
Affine Geometry	1-5
Affine Spaces	1-6
Frames	1-12
Matrix Representations of Points and Vectors	1-14
Affine Transformations	1-15
Matrix Representations of Affine Transformations	1-20
Projective Transformations	1-22
Matrix Representations of Projective Maps	1-24
Euclidian Geometry	1-24
The Inner Product	1-25
Normal Vectors and the Dual Space	1-26
Matrix Representations of Dual Vectors	1-30
A Geometric Abstract Data Type	1-33
General Workings	1-33
Ambiguity Revisited	1-37
A Simple Wire Frame Display Program	1-38
Point Creation	1-39
Clipping	1-40
Transformation to Screen Space	1-43
Scan Conversion	1-44
The Ray-Sphere Intersection Problem	1-46
Implementation Notes	1-48
Summary	1-51
References	1-51
A Brief Review of Linear Algebra	1-53
Selected Procedures of the Abstract Data Type	1-55
Appendix: Slides	1-61
<b>Polar Forms and Splines</b>	2-0
Linear and Quadratic Segments	2-0
Bernstein B'ezier Curve Segments	2-1
Bernstein Polynomials	2-1
A Quadratic Polar Form	2-2
General Multiaffine Polar Forms	2-3
Graphical Representation	2-4
Evaluation, Subdivision, the deCasteljau Algorithm	2-4
Degree Raising	2-6
Derivatives and Continuity	2-6
Progressive Polar Forms	2-7
Basis Splines	2-9
Subdivision and Knot Insertion	2-10

Conversion of B-spline Segments to B'ezier Segments	2-11
References	2-12
<b>Computational Linear Algebra</b>	3-0
Matrices, Vectors, Associated Spaces	3-0
Systems of Linear Equations	3-1
Triangular Forms of Equations	3-3
Fundamental Transformations	3-5
Reduction to Triangular Form	3-9
Constrained Equation Systems	3-12
References	3-12
<b>Constraint Based Curve Manipulation</b>	4-0
Introduction	4-1
Curve and Constraint Notation	4-2
Solving Underdetermined Systems of Equations	4-4
Preprocessing and Iterative Evaluation	4-6
Applying Property Constraints	4-7
Single Constraint Systems	4-8
Double Constraint Systems	4-8
Triple Constraint Systems	4-11
Manipulating Curvature	4-12
Summary and Future Work	4-14
References	4-15
<b>Hierarchical Free-Form Surface Modeling</b>	5-0
Introduction	5-1
Terminology	5-1
Free-Form Models for Articulated Figures	5-4
Hierarchical B-Splines	5-10
Local Refinement and Overlays	5-10
Offset Referencing	5-15
Surface Representation and Storage	5-17
Editing Surfaces Using Offset Referencing	5-23
Multi-level Editing	5-24
An Interactive Editing System	5-27
Edit Points	5-27
Editing Arbitrary Surface Positions	5-30
Summary	5-30
Generalized Hierarchical Surfaces	5-32
Other Bases	5-32
Non-uniform Knot Spacing	5-33
Curves and Volumes	5-33
A Generalized Form	5-34
Offset Fields and Offset Mapping Functions	5-37
Summary	5-41
A Hierarchical Surface Model for Animated Figures	5-42
Extending Offset Referencing	5-42

Attaching Control Nodes	5-43
Multi-segment Joints	5-45
Procedural Methods	5-46
Dynamic Hierarchical Surfaces	5-47
Summary	5-49
References	5-50
Black and White Plates	5-52
<b>Surface Fundamentals</b>	6-0
Vector Calculus	6-0
Differential Curve Geometry	6-1
Differential Surface Geometry	6-7
Surface Characteristics	6-14
Curve Net Fitting	6-17
Sweeping	6-26
References	6-33
<b>Surface Fitting with Hierarchical Splines</b>	7-0
Introduction	7-1
Data Fitting	7-2
Kronecker Products	7-4
Interpolation and Least Squares	7-5
Constraints	7-7
Hierarchical Representation	7-10
References	7-13
<b>Generalized B-spline Surfaces of Arbitrary Topology</b>	8-0
Introduction	8-0
Previous Work	8-1
S-patches	8-1
The <i>N</i> -sided Hole Problem	8-3
Sabin Nets	8-5
Generalized B-spline Schemes	8-6
Generalized Biquadratic Scheme	8-7
Generalized Bicubic Scheme	8-7
Conclusions	8-7
Figures	8-8
References	8-9
Appendix: Slides	8-10
<b>Quality Assessment of Spline Surfaces</b>	9-0
Outline	9-1
Figures	9-2
References	9-3
<b>Application of Color Graphics: Display of Surface Curvature</b>	9-5
Introduction and Background	9-6
The Problem of Surface Curvature	9-6
Approach	9-7
Surface Curvature	9-7

Graphical Representation	9-8
Image Generation	9-9
Automotive Components	9-9
Figures	9-10
Discussion	9-12
Conclusions	9-12
References	9-12
Appendix: Formulas	9-13
<b>Lighting Controls for Synthetic Images</b>	10-0
Introduction	10-1
The Lighting Model	10-2
Light Controls	10-3
Light Direction and Concentration	10-3
Light Flaps and Cones	10-4
Figures	10-5
Light Source Color Blending	10-7
Conclusions	10-8
References	10-9
<b>Color Plates: Hierarchical Surfaces</b>	11-0
<b>Color Plates: Surface Quality</b>	11-2
<b>Rational B-Splines, Properties and Applications</b>	12-0