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COURSE NOTES

C14

FRACTAL MODELING IN 3-D
COMPUTER GRAPHICS AND
IMAGING

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Course Abstract

July 30, 1991. With its 1977 christening, fractal geometry entered into what is now an obvious synergy with computer graphics. The study of fractal shapes has been greatly enhanced by the use of computer graphics to visualize these intricate and ever elusive forms. Fractal geometry, as it becomes better understood — better controlled — is continually repaying its debt to computer graphics in the form of object models. The first of these was a terrain model based on a form of Brownian motion measured across two dimensions. Later contributions have ranged from the novel geometric “monsters” left over from decades past, to trees, flowers and other botanical structures created from simple grammars. This course, an exposition of the synergy between fractal geometry and computer graphics, shows the leading edge in this hybrid of fields.

Speaker Biographies

Charlie Gunn. Mr. Gunn received his M.S. in Mathematics from the University of North Carolina at Chapel Hill in 1983. He worked at Tektronix in 1983-84 and Pixar in 1984-87. From 1988-1990 he held a position at the Geometry Supercomputer Project at the University of Minnesota. There he worked to bring computer graphics into the realm of mathematical research, and vice-versa. Since May, 1991 he continues this research with the newly established Geometry Center at the University of Minnesota. He is particularly interested in hyperbolic geometry, dynamics, and how computers can support mathematical intuition. Mr. Gunn is a member of the ACM, SIGGRAPH and the AMS.

John C. Hart. Mr. Hart is a Ph.D. "wannabe" in the Electrical Engineering and Computer Science Department of the University of Illinois at Chicago. He received his M.S. in Computer Science at UIC in 1989 and his B.S. in Computer Science at Aurora University in 1987. Since 1988, he has been a member of the Electronic Visualization Laboratory at UIC where he researches the modeling, rendering and animation of 3-D deterministic fractals. He interned at the IBM T.J. Watson Research Center in 1989 and at AT&T Pixel Machines in 1990. He is currently funded under a UIC Graduate College student fellowship. He will serve his post-doctorate at the EVL in 1991-92 and will become a father in October 1991.

Benoit B. Mandelbrot. Dr. Mandelbrot is an IBM Fellow at the T.J. Watson Research Center at Yorktown Heights, NY and the Abraham W. Robinson Professor of Mathematics at Yale University. His classic "Fractal Geometry of Nature" (W.H. Freeman, 1982) established the field of fractal geometry. His research is concentrated on extreme and unpredictable irregularity in natural phenomena in the physical, social and biological sciences. He belongs to the National Academy of Sciences and the American Academy of Arts and Sciences.

F. Kenton Musgrave. Mr. Musgrave has been a research assistant to Benoit Mandelbrot in the Mathematics Department of Yale University since 1987. He received his M.S. in computer science from the University of California, Santa Cruz in 1987. Mr. Musgrave is currently a lecturer and Ph.D. candidate in the Yale Department of Computer Science. His background is in fine arts and physical sciences, and his research is primarily in the area of modeling natural phenomena, specifically synthetic imagery of fractal landscapes. Mr. Musgrave considers the main import of his work to be in the realm of fine art. He is a contributor to the SIGGRAPH art show, to various art exhibits in the United States and abroad, and participated in the production of the "New York Notes" performance at the Guggenheim Museum in 1990 and at the Alice Tully Hall of the Lincoln Center in 1991. Mr. Musgrave is a member of the ACM, SIGGRAPH and the IEEE Computer Society.

V. Alan Norton. Dr. Norton was born in Utah. He received his B.A. degree from the University of Utah in 1968 and his Ph.D. in mathematics from Princeton in 1976. He served on the mathematics faculty of University of Utah and Hamilton College, then became interested in computer graphics while consulting at Evans and Sutherland Computer Co. In 1980 he joined the IBM T.J. Watson Research Center, working with Benoit Mandelbrot on the computation and visualization of fractals. In 1982 Norton initiated an investigation of architectures for parallel processing which led to the founding in 1984 of the RP3 Research Parallel Processing Project. In 1987, Norton established a project at IBM in computer animation research in the area of fractals and physically-based modeling. Norton is the Courses Chair for SIGGRAPH '92.

Przemyslaw Prusinkiewicz. Dr. Prusinkiewicz is Associate Professor of Computer Science at the University of Regina. He holds an M.S. and Ph.D., both in Computer Science, from the Technical University of Warsaw. Before joining the faculty of the University of Regina, he was Assistant Professor at the University of Science and Technology of Algiers. Professor Prusinkiewicz is co-author of three textbooks and two books: "Lindenmayer Systems, Fractals and Plants" (Springer-Verlag 1989) and "The Algorithmic Beauty of Plants" (Springer-Verlag 1990), as well as over 40 technical papers. He was a Visiting Professor at Yale University (1988) and at L'Ecole Polytechnique Federale de Lausanne (1990), and an invited researcher at the University of Bremen (1989). Dr. Prusinkiewicz is a member of the ACM, SIGGRAPH and the IEEE Computer Society.

Dietmar Saupe. Dr. Saupe received his Dr. rer. nat. in Mathematics, 1982 at the University of Bremen. He was a Visiting Assistant Professor of Mathematics at the University of California, Santa Cruz in 1985-87 and since 1987 he is an Assistant Professor of Mathematics at the University of Bremen. There he is a researcher at the Dynamical Systems Graphics Laboratory with main interests in mathematical computer graphics and experimental mathematics. He has been involved in several past SIGGRAPH courses on fractals, as the course chair in 1988 and 89 and as coauthor/coeditor of "The Science of Fractal Images," 1988, and "Fractals for the Classroom," 1990, both from Springer-Verlag. He is one of the contributors to the exhibit "Frontiers of Chaos" which is being shown worldwide under the auspices of the Goethe-Institute. Dr. Saupe is a member of the ACM and SIGGRAPH.

Charles Wuorinen. Mr. Wuorinen was composer in-residence and new music advisor to the San Francisco Symphony from 1984-89. Charles Wuorinen has received two Guggenheim awards, three Rockefeller Foundation grants, commissions from the Ford, Fromm and Koussevitsky foundations and several grants and commissions from the National Endowment for the Arts and the New York State Council on the Arts. The composer won the Pulitzer Prize in 1970 for his electronic work "Time's Encomium." The founder of the Group for Contemporary Music, Charles Wuorinen is a member of the American Academy of Arts and Letters and his works have been recorded on nearly a dozen labels. He is presently working on a new score for the New York City Ballet.

Acknowledgements

We would first like to acknowledge the support of Dietmar Saupe and Przemyslaw Prusinkiewicz, without whose initial encouragement, this course would have never gotten off the ground. Benoit Mandelbrot has also been an active participant in the formulation of this course.

The section on fractal image compression is largely due to Laurie Reuter. She was instrumental in locating some of the finest resources in such a secretive area of science. Two major manuscripts on the subject are reproduced here due to the great generosity of their authors: Arnaud Jacquin and Edward Vrscay.

Furthermore, the information on the Cubic Connectedness Locus in these notes and in Charlie's talk was possible only with the help of two of its most accomplished investigators: John Milnor and John Hubbard.

Credit is due to John's Patty and Ken's Lisa for their support and for the many lonely hours they have endured because of this course. The computer widow is to be pitied, but the SIGGRAPH widow truly deserves sympathy.

We would like to thank Craig Kolb and Bill Etra for working hard on all manner of things that we put aside in favor of this course, and to Howard Stokar for his patience and perseverance in arranging for the recording of New York Notes.

These course notes were prepared using the facilities of the Electronic Visualization Laboratory at the University of Illinois at Chicago. We owe a great debt of gratitude to its co-directors: Tom DeFanti and Dan Sandin, and its assistant director: Maxine Brown.

Finally we would like to thank SIGGRAPH '91 courses chair Rich Ehlers and committee members Mark Lee, Alan Norton and Dino Schweitzer for their expert guidance through the perils and pitfalls that accompany course organization.

J.C.H.
Chicago, IL

F.K.M.
New Haven, CT

Course Schedule

Opening	8:30
John Hart and Ken Musgrave	
Part I: Random Fractal Models	
Fractional Brownian Motion and Models of Nature	8:45
Ken Musgrave	
Break	10:15
Part II: Linear Deterministic Fractal Models	
Automata, Languages and Iterated Function Systems	10:30
Przemyslaw Prusinkiewicz	
Linear Fractals in 3-D Computer Graphics	11:00
John Hart	
Lunch	12:00
Part III: Non-Linear Deterministic Fractal Models	
Julia Sets and Mandelbrot Sets in 3-D	1:00
Alan Norton	
Visualizing the Cubic Connectedness Locus	2:00
Charlie Gunn	
Defining and Rendering Deterministic Fractal Implicit Surfaces	2:30
Dietmar Saupe	
Break	3:00
Part IV: Applications	
Fractal Image Compression	3:15
John Hart	
New York Notes	4:00
Benoit Mandelbrot and Charles Wuorinen	
Closing	5:00

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