

**SIGGRAPH 1991**  
**18th International Conference**  
**On Computer Graphics and**  
**Interactive Techniques**

**Las Vegas Convention Center**  
**28 July - 2 August**

## **COURSE NOTES**

## **C2**

### **MATH FOR SIGGRAPH**

***Chair***

**Ken Shoemake**  
**Otter Enterprises, Inc.**

***Lecturers***

**Tony DeRose**  
**University of Washington**  
**Michael Kass**  
**Apple Computer, Inc.**  
**Tom Sederberg**  
**Brigham Young University**  
**Frances Yao**  
**Xerox PARC**

# Table of Contents

Schedule .....	i
Introduction.....	ii
Speakers .....	iii
<b>Tony DeRose</b>	
Coordinate-Free Geometric Programming.....	A-1
<b>Ken Shoemake</b>	
How to Turn Right .....	B-1
Animating Rotation with Quaternion Curves .....	B-8
Quaternion Calculus for Animation.....	B-18
ARCBALL.....	B-37
Polar Decomposition for Rotation Extraction.....	B-44
<b>Frances Yao</b>	
Tutorial on Computational Geometry.....	C-1
A Whirlwind Tour of Computational Geometry .....	C-8
Efficient Binary Space Partitions for Hidden-Surface Removal and Solid Modeling.....	C-24
Provably Good Mesh Generation.....	C-43
<b>Tom Sederberg</b>	
Algebraic Methods.....	D-1
<b>Ken Shoemake</b>	
Digital Transforms and Filters.....	E-1
Convolution Surfaces .....	E-27
Simplicit: a Simple Implicit Surface Polygonizer .....	E-33
<b>Michael Kass</b>	
Introduction to Continuum Dynamics for Computer Graphics.....	F-1
Rapid, Stable Fluid Dynamics for Computer Graphics .....	F-15
GO: A Graphical Optimizer.....	F-24

# Math for SIGGRAPH

## Approximate Schedule

<b>Start</b>	<b>Time</b>	<b>Speaker</b>	<b>Topic</b>
8:30	0:15	Shoemake	Introduction and Overview
8:45	1:15	DeRose	Affine, Euclidean, and Perspective Geometry
10:00	0:15	—	Break
10:15	0:45	Shoemake	Quaternions and Rotations
11:00	1:00	Yao	Computational Geometry
12:00	1:30	—	Lunch
1:30	1:00	Sederberg	Algebraic Geometry
2:45	1:00	Shoemake	Digital Transforms and Filters
3:30	0:15	—	Break
3:45	1:15	Kass	Numerical Techniques for Modeling and Dynamics
5:00	—	—	END

## Introduction

Welcome to the second edition of Math for SIGGRAPH. It is a pleasure for me to be able to once again offer you an excellent selection of speakers and topics. I hope you will enjoy the course, and profit from these notes afterwards.

My motivation for creating this course was simultaneously selfish and altruistic: I wanted to extend my education and yours by assembling the kind of course I wanted for myself. Mathematics is an international endeavor extending over centuries, and its breadth and depth exceed the capacities of even the most brilliant of researchers. While there is truly no substitute for getting your hands dirty, it helps considerably to hear a subject explained by someone who knows it and loves teaching it. There are fundamental connections between topics that appear different on the surface, else a course as broad as this would be hopeless.

So, what is fundamental? Algebra, especially linear algebra, goes at the top of my list. Every topic covered in this course uses linear algebra: matrices, vectors, and sometimes calculus (calculus itself is based on linear approximations to curves). It is important to understand linear algebra both abstractly—without coordinates, and concretely—with all the numerical techniques required for computation. Moving on, it is important to know other algebras, especially those connected with polynomials. Most of this algebraic theory is simple, and will get you through 90% of the material in this course—and 90% of applied mathematics in general. The remaining 10% involves geometry, number theory, and a few other odds and ends.

We are not going to teach you these fundamentals; this is, after all, an intermediate course. We are going to use them over and over again, until you can hardly doubt their utility and importance. If you want to study the fundamentals more on your own, I recommend Gilbert Strang's *Introduction to Applied Mathematics*. We also are not going to make you experts in a day, in any of the topics we will cover. We are going to try to give you the "lay of the land," a feel for how things fit together and what's important. We can do no more, since, ultimately, as I said before, you only really learn material when you try to use it. The speakers will conclude their talks with some of their current research, which makes it more interesting for them, and for you, too (I hope).

Although the course only lasts a day, preparations began many months ago. Thanks to all the people—speakers, course committee, A/V crew—who contributed.

## About the Speakers

Tony DeRose (the students' favorite in 1989) has advanced our understanding of geometric continuity, Bézier simplex composition, multi-sided patches, and so on. Michael Kass is known for his work in computer vision and dynamics simulations (and for juggling clubs with his feet). Tom Sederberg (the 1991 technical program chair) brought the computer graphics use of algebraic surfaces beyond quadrics with his seminal work on implicitization, and represents a commitment to quality teaching. Ken Shoemake, the chair (as in 1989), made popular the use of quaternions for describing and animating rotations, and in earlier years worked in computer music at the Stanford Center for Computer Research in Music and Acoustics. Frances Yao is a leader in computational geometry, and co-author of a recent survey of the field.

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