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COURSE NOTES

25

UNIFYING PARAMETRIC AND IMPLICIT SURFACE REPRESENTATIONS

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1 Summary

Parametric and implicit surface representations are each suited to certain types of problems in computer graphics. This course will compare, contrast, and unify these approaches with an emphasis on the Bernstein/Bézier representation.

2 Course Description

In computer graphics, techniques have been developed for representing and rendering scenes of increasing complexity. One idea that has proven to be fairly effective is to represent a surface using polynomials or ratios of polynomials. Two approaches for surface representation incorporating this idea are

The Parametric Representation

$$x = f_1(s,t)$$
$$y = f_2(s,t)$$
$$z = f_3(s,t)$$

where f_1, f_2, and f_3 are polynomials in s and t or ratios of polynomials in s and t.

The Implicit Representation

$$g(x,y,z) = 0$$

where g is a polynomial in x, y, and z. This course will begin with a brief review of both representations and their relative strengths and weaknesses with regard to operations such as model construction, shape control, intersection, rendering, and finite element mesh generation.

Note that for both representations we are restricting our attention to polynomials or ratios of polynomials. A given polynomial can be represented in various forms. In both approaches, the selection of a particular representation is fundamental. For example, in the univariate case, one straightforward possibility would be as linear combination of powers:

$$q(u) = \sum_{i=0}^{n} a_i u^i$$

However, this representation suffers from the deficiency that there is no intuitive relationship between the coefficients a_i and the corresponding graph of the polynomial $q(u)$. To address this shortcoming, we will introduce the Bernstein/Bézier representation of polynomials. Using this representation for $q(u)$ engenders coefficients that relate to the graph of $q(u)$ in a more predictable manner. This course will specifically focus on the Bernstein/Bézier representation of polynomials in two and three variables.

After an introductory lecture on the Bernstein/Bézier representation of polynomials, we will apply this material to three problems that are fundamental to surface design and manipulation. We will view each problem from both the perspective of parametric and implicit representations.

- **Piecewise Surface Generation**

  To introduce parametric piecewise surfaces, we will first show how to smoothly join parametric Bérnstein/Bézier curves and then explain how to use this approach to form smooth tensor product parametric Bernstein/Bézier surfaces.

  This will then be expanded upon by a discussion of triangular parametric Bernstein/Bézier surfaces and their n-sided generalization known as S-patches.

  The implicit portion of this section will present techniques for generating the lowest degree implicit surface patches that smoothly contain any given number of points and space curves, of arbitrary degree. This method allows for interpolation of point and curve data with prescribed normal data. Interactive shape control is achieved by representing the resulting families of implicit surface patches in their Bernstein/Bézier form and adjusting ordinates on the Bézier control net.

- **Rendering Methods**

  This section will focus on methods for rendering parametric and implicit surfaces with an emphasis on adaptive methods. We will begin by considering the problem of displaying a rational space curve. We will discuss techniques for stepping along the space curve with the step size being adaptively chosen based on the curvature and torsion of the curve. Since parametric surfaces are covered by two independent families of rational space curves, we may use this method for space curves to derive an adaptive rectangular tiling of the surface. This rectangular tiling can be enhanced to a uniform triangulation at trimming boundaries of adjacent surface patches so as to avoid cracks.

In the case of the implicit representation, we will discuss several recently developed techniques for directly ray tracing implicit surfaces. These techniques will focus on the use of the Bernstein/Bézier representation in the ray/surface intersection problem. We will also consider methods for adaptively tiling an implicit surface using polygons. This methods will involve various schemes for recursively subdividing space.

- **Conversion Methods**

  This portion of the course will begin with some basic facts concerning conversion methods. For example, some curves and surfaces do not have a parametric form, but every parametric curve or surface has an implicit form.

  The next portion will discuss methods for converting from the implicit form to the parametric form. Topics will include special methods for quadratic curves and surfaces as well as methods for cubic curves. Geometric and algebraic methods of conversion and their relationship to each other will be discussed.

  The third part will address the philosophical question: "does every parametric surface really have an implicit form?" and, if time permits, will illustrate why this question should be asked using an example involving offset curves and surfaces. Two approaches to this problem will be investigated: The first involves Sylvester's resultant from classical elimination theory while the second uses ideal theory and Gröbner bases.

  The final portion will address numerical and practical issues concerning conversion. Topics include exact arithmetic versus floating-point implementations as well as some experimental results with standard implementations of Gröbner basis methods and Sylvester resultant methods.

3 Outline and Schedule

- Univariate Bernstein Polynomials (Warren, 25 minutes)
- Parametric Bernstein/Bézier Curves and Tensor Product Surfaces (Barsky, 45 minutes)
- Multivariate Bernstein Polynomials (Warren, 20 minutes)
- Break (30 minutes)
- Triangular Parametric Bernstein/Bézier Surfaces and S-patches (DeRose, 45 minutes)
- Implicit Algebraic Surface Patch Generation using Hermite Interpolation (Bajaj, 45 minutes)
- Lunch (60 minutes)
- Rendering Techniques for Parametric Surfaces (Bajaj, 45 minutes)
- Rendering Techniques for Implicit Surfaces (Warren, 45 minutes)
- Break (30 minutes)
- Conversion Methods between Parametric and Implicit Representations (Hoffmann, 90 minutes)

4 Who Should Attend

This course is intended for those interested in state-of-the-art techniques in computer-aided geometric design and modeling. Typical participants might be researchers and practitioners in geometric design and modeling or software engineers developing 3D modeling systems. This course will assume some basic familiarity with Bernstein/Bézier curves.

5 Recommended Background/Difficulty

This is an advanced course and will assume some basic familiarity with Bernstein/Bézier curves.

Note regarding textbooks for this course

This course presents concepts and techniques that unify implicit and parametric representations of curves and surfaces. The methods we present here rely on diverse facts and on a rather broad background in parametric and implicit curves and surfaces. We know of no single book that provides this background in all aspects. However, parametric curves and surfaces are very well covered in the book by Bartels, Beatty, and Barsky, *An Introduction to Splines for Use Computer Graphics and Geometric Modeling.* Implicit curves and surfaces, and conversion techniques between them, are presented in detail in Hoffmann's book, *Geometric and Solid Modeling.*

We wanted to include both books with these notes. Unfortunately, this was impossible for logistical reasons; thus, we decided to include the book by Bartels, Beatty, and Barsky, and secure for you a discount coupon for Hoffmann's book.

The chapter headings for Hoffmann's book is provided on the coupon. Chapter 5 (Representation of Curved Edges and Faces), Chapter 6 (Surface Intersections), and Chapter 7 (Gröbner Bases Techniques) present the relevant material on implicits, on conversion between implicits and parametrics, and on useful techniques from abstract algebra and algebraic geometry. Each chapter begins with a tutorial that explains, in simple terms, why mathematicians have developed certain esoteric concepts and methods from time to time. These introductions also clarify why some of the methods are relevant in applied situations. Such mini-tutorials are possible because much of what is presented by mathematics in the most abstract terms is often motivated by very concrete, down-to-earth problems and ideas.

The mathematician abstracts because he or she desires maximum generality, precision, and economy of expression. Applications, on the other hand, are specific, and not all the generality the mathematician wants to achieve will be needed. Thus, the chapter introductions link the mathematicians world with the practitioner's world of concrete problems and experiences, and can therefore function as "road maps". Indeed, these introductions are used in this manner in the balance of each chapter, where specific mathematical techniques are richly illustrated by examples and applications.