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Course Notes 23

COMPUTATIONAL REPRESENTATIONS OF GEOMETRY

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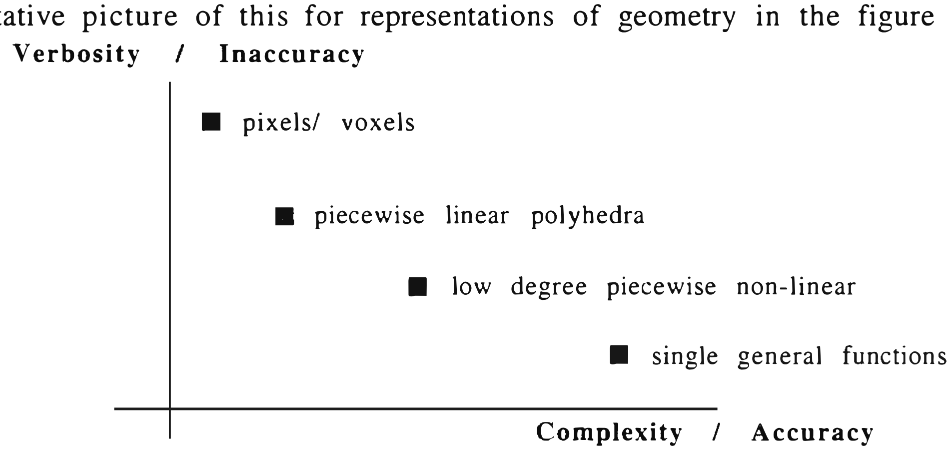
Computational Representations of Geometry

Bruce Naylor

Introduction

Computational representations of geometry provide the foundations for all the various areas of computing that involve geometry. These areas currently include Computer Graphics, Computer-Aided Geometric Design, Scientific Visualization, Computational Geometry, Finite Element Analysis, Robotics, Computer Vision and Image Processing. Yet only a few representations of geometry have emerged to date. We believe that the reason for this is that geometric sets are describable in terms of only a few fundamental aspects, e.g. their topology, or set theoretic structure, etc. Many of the representations, in their purest form, describe primarily a single aspect of geometric sets. Each "pure" representation is the language for expressing that aspect. For example, the topology of a set can expressed by what we naturally call "a topological representation" of the following form: a graph with a one-to-one correspondence between graph nodes and topologically connected components, and where graph edges indicate incidence between components. In addition to pure representations, other types of representations may be hybrids, combining multiple aspects simultaneously.

Representations of geometry have a very strong connection to "traditional" mathematics, both historically, dating back to Classical Greek civilization, and to modern subjects. These include Set Theory, Graph Theory, Algebraic Topology, etc., in addition to the various varieties of Geometry: Projective, Analytic, Algebraic, Differential, and Combinatorial. What is different, as everyone knows, is that computation is "constructive mathematics", and the primary measure of value is not provability per se (as much as we might want error free programs), but rather performance and accuracy. The pursuit of efficiency has led numerous times to what might initially be considered as a counter-intuitive result: that computing with many simple pieces can be faster than attempting to process fewer but more computationally complicated pieces. This is what we call the verbosity/complexity tradeoff: we trade a relatively small number of complicated operations for many more but simpler operations. This is a very important consideration in geometric computation, and we have given a qualitative picture of this for representations of geometry in the figure below.



Verbosity / Complexity Tradeoff for Geometric Representations