



SIGGRAPH 1994

*21st International Conference
On Computer Graphics and
Interactive Techniques*

*Orange County Convention Center
Orlando, Florida
July 24-29*

Course Notes

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NEW DIRECTIONS FOR
FRACTAL MODELING IN
COMPUTER GRAPHICS

Organizer

John C. Hart
Washington State University

Lecturers

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The George Washington University

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Richard F. Voss
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Course Overview

Although its origins can be traced back to the beginning this century, the Mandelbrot set launched Fractal Geometry to the peak of its popularity (and hype) in the 1980's, fueled by the controversy of arguments over creased mountains and reports of 10,000-to-1 compression rates. Much has changed since then.

Now there seem to be a multitude of programs for exploring the Mandelbrot set, and with recent results on its local connectedness and the dimension of its boundary, “the most complicated thing in mathematics” appears to hold very little mystery anymore. The iterated function system, which could cause a fern leaf to magically appear out of nowhere, is now maturing into a practical geometric representation. Though still protected by patent #5,065,447, fractal image compression is no longer a mystery, and now variations on its theme are researched extensively. L-systems have evolved from graphpal cartoon trees and 2-D turtle graphics into a sophisticated fractal language of botany. Midpoint subdivision mountain algorithms have been replaced with sophisticated multifractal terrain models. In short, the field of fractal geometry has changed, and is in a position to serve computer graphics as computer graphics served it many years ago.

There are two key phrases in the title of this course. The first is *new directions*. We have sacrificed polished tutorial introductions¹ for recent, cutting-edge research contributions in the hope that these notes will inspire and foster new research in fractal geometry. The SIGGRAPH course attendee should walk away with a grasp of the current state-of-the-art in fractal modeling, and plenty of ideas for contributing to this field of research.

The second key phrase is *for computer graphics*. The speakers have all made contributions

¹For which there are several good books, such as:

- Peitgen & Richter, “The Beauty of Fractals,” Springer-Verlag, 1985.
- Peitgen & Saupe (Eds.), “The Science of Fractal Images” Springer-Verlag, 1988.
- Barnsley, “Fractals Everywhere,” Academic Press, 1988.
- Falconer, “Fractal Geometry: Mathematical Foundations and Applications,” Wiley, 1990.
- Prusinkiewicz & Lindenmeyer, “The Algorithmic Beauty of Plants,” Springer-Verlag, 1990.
- Peitgen, Jurgens & Saupe, “Fractals for the Classroom” (2 vols.), Springer-Verlag, 1992.
- Peitgen, Jurgens & Saupe, “Chaos and Fractals,” Springer-Verlag, 1992.

to the development of fractals as a tool for computer graphics, as opposed to the use of computer graphics as a tool to investigate fractals.

The course begins with a discussion of recurrent iterated function systems as a geometric representation, integrating them into mainstream computer graphics through the use of constructive solid geometry. This is followed by a discussion of L-systems — the fractal representation most popular for modeling plants, and a method for converting a language-restricted (recurrent) iterated function systems into an L-system. The rest of the morning is devoted to fractal image compression, first with a specific overview of the original block-coding technique and its many recent improvements, followed by a more general discussion of the inverse problem of fractal geometry and how fractal image compression helps us solve it.

The afternoon begins with a short presentation on the interactive visualization of quaternion Julia sets, and chaotic systems in general. The rest of the afternoon is devoted to random fractals, beginning with an introduction of basic algorithms for fractional Brownian motion — the foundation of fractal terrain models, followed by a discussion of multifractals — shapes of varying dimension. The course concludes with a report on the success of fractal models applied in the other sciences.

Schedule

Topic	Speaker	Time
Open	John C. Hart	8:30
Recurrent Iterated Function Systems	John C. Hart	8:40
L-systems	Przemyslaw Prusinkiewicz	9:30
Break	—	10:30
Fractal Image Compression	Dietmar Saupe	10:45
The Inverse Problem	John C. Hart	11:45
Lunch	—	12:00
Quaternion Julia Sets	John C. Hart	1:30
Random Fractals	Dietmar Saupe	2:00
Multifractals	F. Kenton Musgrave	3:00
Break	—	3:30
Applications	Richard F. Voss	3:45
Close	John C. Hart	5:00

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<i>John C. Hart</i>	
Language Restricted Iterated Function Systems, Koch Constructions and L-systems	4-1
<i>Przemyslaw Prusinkiewicz and Mark Hammel</i>	
A Guided Tour of the Fractal Image Compression Literature	5-1
<i>Dietmar Saupe and Raouf Hamzaoui</i>	
A Discussion of Fractal Image Compression	6-1
<i>Yuval Fisher</i>	
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<i>Yuval Fisher</i>	
Sample Code	8-1
<i>Yuval Fisher</i>	
Fractal Image Compression and the Inverse Problem of Recurrent Iterated Function Systems	9-1
<i>John C. Hart</i>	

Interactive Visualization of Quaternion Julia Sets	10-1
<i>John C. Hart, Louis H. Kauffman and Daniel J. Sandin</i>	
Random Fractals in Image Synthesis ²	11-1
<i>Dietmar Saupe</i>	
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<i>Richard F. Voss</i>	
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<i>Richard F. Voss and James C.Y. Watt</i>	

²This article does not appear in the CD-ROM version of the **course notes**

Speaker Biographies

John C. Hart is an Assistant Professor in the School of Electrical Engineering and Computer Science at Washington State University. Hart received his B.S. in Computer Science from Aurora University, and his M.S. and Ph.D. in Computer Science from the University of Illinois at Chicago. There, he studied fractals in computer graphics as a research assistant in the Electronic Visualization Laboratory, and in 1992, as a postdoctoral research associate jointly with the National Center for Supercomputing Applications. As a graduate student, he interned summers in Alan Norton's group at the IBM T.J. Watson Research Center, and at AT&T Pixel Machines (R.I.P.), and recently consulted for Kleiser-Walczak on the "Luxor" project.

Dr. Hart's research interests currently focus on the relationship between fractal geometry and geometric design, producing both new techniques for implicit surface modeling as well as extending the geometric theory for fractal modeling. He has produced four animations based on his fractal modeling algorithms, one of which (unNatural Phenomena) appeared in the SIGGRAPH '91 Electronic Theater, ABC's "Prime Time Live" and Miramar's "Beyond the Mind's Eye."

With Ken Musgrave, Hart co-organized an advanced fractals course for SIGGRAPH '91, and has since lectured in SIGGRAPH courses on fractals and implicit surfaces. Hart is a member of the ACM and SIGGRAPH, has guest edited special issues of the Communications of the ACM and the SIGGRAPH Video Review, and was a member of the SIGGRAPH '94 Electronic Media Committee.

Ken Musgrave teaches in the Department of Electrical Engineering and Computer Science at The George Washington University. His research is primarily in the area of modeling natural phenomena, specifically realistic imaging of fractal landscapes; however, he considers the production of fine art with the computer to be his primary goal. He received his Ph.D. in computer science from Yale University, where he was a research assistant to Benoit Mandelbrot in the Mathematics Department since 1987.

Dr. Musgrave's research papers have appeared in IEEE Computer Graphics and Applications, the proceedings of SIGGRAPH '89 and Graphics Interface '89, Graphics Gems II and III, the IBM Journal of Research and Development, and other technical publications. His images have been displayed in the SIGGRAPH '88 and '91 Art Shows and have appeared on the covers of "The Science of Fractal Images." IEEE Computer Graphics and Applications, Computer Graphics World, and SunWorld, as well as in National Geographic, "Fundamen-

tals of Interactive Computer Graphics,” the Communications of the ACM and numerous magazines and books internationally.

Przemyslaw Prusinkiewicz is a Professor of Computer Science at the University of Calgary, Canada. Born and raised in Poland, he received his MSc in 1974 and PhD in 1978, both from the Technical University of Warsaw. Before moving to Canada in 1982, he was an Assistant Professor of Mathematics at the University of Science and Technology in Algiers. In 1988 he was a Visiting Assistant Professor of Mathematics and Computer Science at Yale University.

The focus of Dr. Prusinkiewicz’s research is in the area of fractals, and the modeling and visualization of biological structures. He is coauthor of the books “Lindenmeyer Systems, Fractals, and Plants” (with James Hanan, 1989) and “The Algorithmic Beauty of Plants” (with Aristid Lindenmeyer, 1990), both published by Springer-Verlag. Dr. Prusinkiewicz is a member of the ACM, SIGGRAPH, and the IEEE Computer Society, and was a member of the SIGGRAPH ’94 paper’s committee.

Dietmar Saupe was born in 1954 in Bremen. Dr. rer. nat. in Mathematics 1982, at the University of Bremen. Visiting Assistant Professor of Mathematics at the University of California, Santa Cruz, 1985-87, and from 1987 to 1993, Assistant Professor of Mathematics at the University of Bremen, where he was a researcher at the Center for Complex Systems and Visualization. He is now at the Institute für Informatik at the Universität Freiburg, Germany.

Dr. Saupe’s interests are in mathematical computer graphics, experimental mathematics, and scientific visualization. He has been involved in several past SIGGRAPH courses on fractals, and has chaired them from 1988 to 1991. He is coauthor/coeditor of “The Science of Fractal Images,” 1988, and “Fractals for the Classroom” 1991/92, both from Springer-Verlag. He is one of the contributors to the exhibit “Frontiers of Chaos,” which was shown worldwide under the auspices of the Goethe-Institute. Dr. Saupe is a member of the ACM and SIGGRAPH.

Richard F. Voss is an internationally recognized physicist and popular lecturer on fractals. He has presented over 120 major invited lectures on fractal geometry and has published over 75 scientific articles.

Born in 1948 in Minnesota, he received a B.S. degree in physics from M.I.T in 1970 and a National Science Foundation Fellowship for graduate studies. After receiving a Ph.D. in physics from the University of California at Berkeley in 1975 he joined the IBM Research Division as a Research Staff Member. At IBM he worked closely with Dr. Benoit Mandelbrot (the “father” of fractals) and continued his research in condensed matter physics. His mastery of scientific computer graphics has been instrumental in the rapid acceptance of fractals as a useful language. His computer generated images have appeared widely in numerous magazines, books, television shows, IBM commercials, and on the covers of *The Fractal Geometry of Nature* and *The Turbulent Mirror*. He is a co-author of the book *The Science*

of *Fractal Images* published by Springer-Verlag in 1987 and he was chairman of the 1988 Gordon Research Conference of Fractals.

His research in condensed matter physics includes low temperature macroscopic quantum phenomena in Josephson junction devices, $1/f$ noise, and the fractal characteristics of random media and growth processes. He has been elected a Fellow of the American Physical Society, to membership in Sigma Xi, and he has received major IBM internal awards for his work on macroscopic quantum tunneling and on random fractals. He has had sabbatical assignments at U.C. Santa Barbara and Harvard University. He has been honored as the 1987-8 Siefert Memorial Lecturer at Montana State University, a 1987-8 Distinguished Lecturer at Trinity University, a 1990 Distinguished Lecturer at Utah State University, 1992 Hayes lecturer at Oberlin and a visiting professor at the Universities of Bremen and Oslo. In 1993 he was elected Professor of Applied Physics, adjunct, at Yale University while continuing association with IBM Research as a visiting scientist. He is currently teaching a Yale undergraduate course on Fractal Geometry and is working on a book about Chaos, Fractals, and Art with the NY sculptor, Rhonda Shearer.

Acknowledgements

I would first thank the speakers for investing the sizable time and effort required for a SIGGRAPH course.

Przemek Prusinkiewicz, the last fractals course organizer, hosted a wonderful visit for me to the University of Calgary, and I appreciate his guidance then and since on the content of this course. In the same spirit, Dietmar Saupe, another fractals course organizer, offered yet more helpful suggestions. Mike Bailey and the SIGGRAPH '94 courses committee, especially our coordinator, Scott Senften, guided this course to completion with a continuous supply of deadlines, forms and e-mail.

Both Dietmar Saupe and I are grateful to Yuval Fisher for giving us permission to use his introduction to fractal image compression from Appendix A of "Chaos and Fractals" (Peitgen, Jürgens, Saupe Eds.) Springer-Verlag, 1992, and for allowing us to use Chapter 3 and Appendix A of his new book "Fractal Encoding — Theory and Application to Digital Images", Springer-Verlag, 1994. Dietmar and I are also grateful to Carl Evertsz for giving us permission to edit and use his and Benoit Mandelbrot's introduction to multifractals from the Appendix B of "Chaos and Fractals" (Peitgen, Jürgens, Saupe Eds.) Springer-Verlag, 1992.

These notes were printed using the facilities of the Imaging Research Laboratory and of the School of Electrical Engineering and Computer Science, at Washington State University.

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