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Course Notes

11

WAVELETS AND THEIR
APPLICATIONS IN
COMPUTER GRAPHICS

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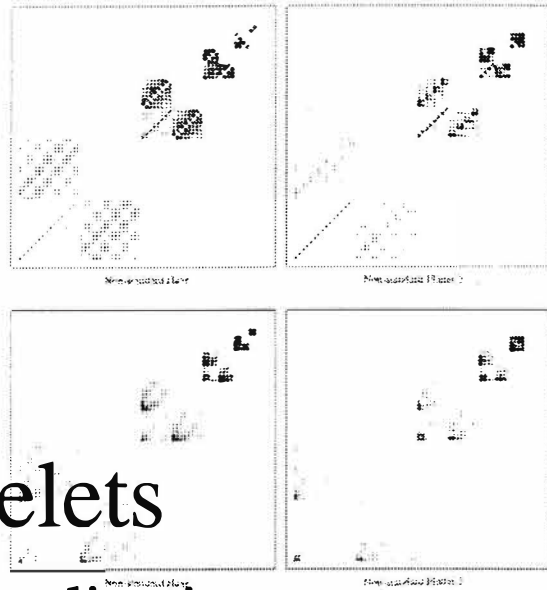
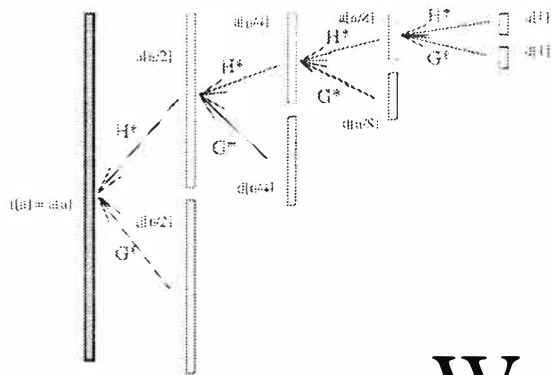
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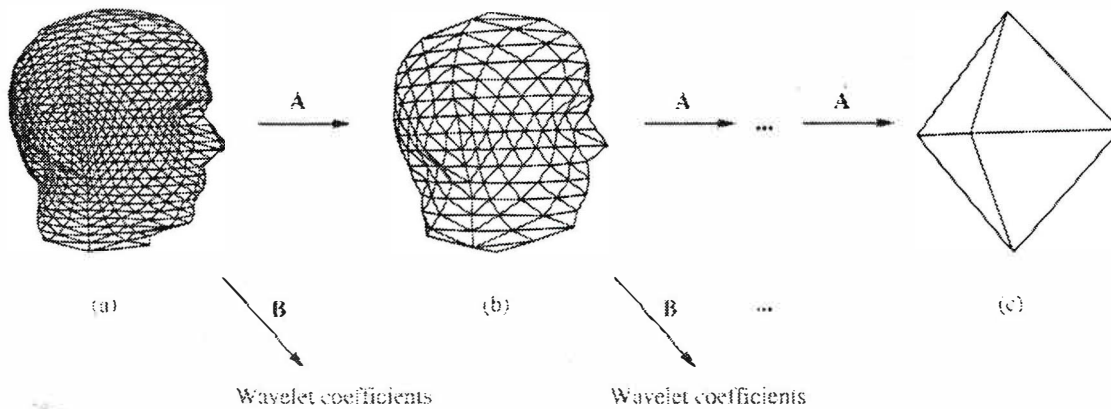
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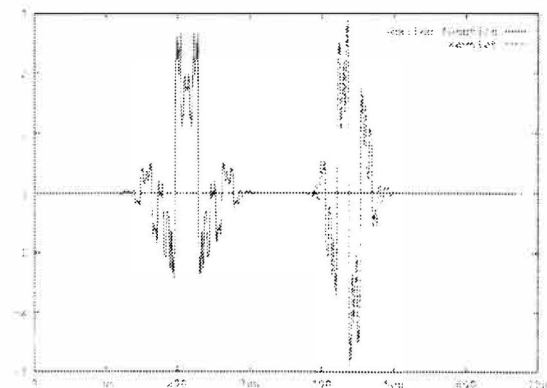
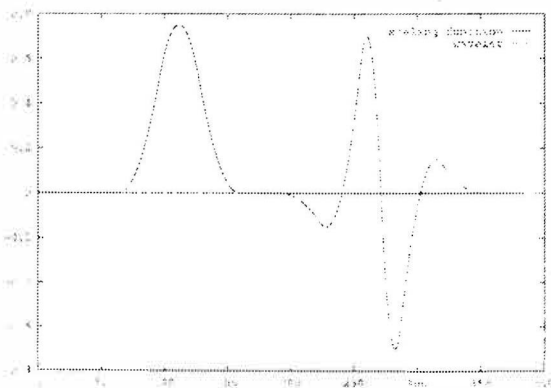
SIGGRAPH '94 Course Notes



Wavelets and their Applications in Computer Graphics



Organizer:
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*Nothing to do with sea or anything else.
Over and over it vanishes with the wave.*

– Shinkichi Takahashi

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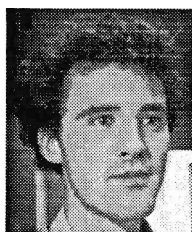
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Table of Contents

Preamble – Alain Fournier	1
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1 Prolegomenon	1
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I Introduction – Alain Fournier	3
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1 Scale	3
1.1 Image pyramids	3
2 Frequency	5
3 The Walsh transform	6
4 Windowed Fourier transforms	8
5 Relative Frequency Analysis	11
6 Continuous Wavelet Transform	11
7 From Continuous to Discrete and Back	12
7.1 Haar Transform	12
7.2 Image Pyramids Revisited	14
7.3 Dyadic Wavelet Transforms	17
7.4 Discrete Wavelet Transform	18
7.5 Multiresolution Analysis	19
7.6 Constructing Wavelets	19
7.7 Matrix Notation	21
7.8 Multiscale Edge Detection	22
8 Multi-dimensional Wavelets	22
8.1 Cartesian Product	22
8.2 Non-standard Basis	22
8.3 Quincunx Scheme	22
9 Applications of Wavelets in Graphics	23
9.1 Signal Compression	23
9.2 Modelling of Curves and Surfaces	26
9.3 Radiosity Computations	26
10 Other Applications	26

1	Introduction	31
1.1	A recipe for finding wavelet coefficients	31
1.2	Wavelet decomposition	33
1.3	Example of wavelet decomposition	34
1.4	From the continuous wavelet transform to more compact representations	35
2	Multiresolution: definition and basic consequences	35
2.1	Wavelet spaces	37
2.2	The refinement equation	38
2.3	Connection to filtering	39
2.4	Obtaining scaling functions by iterated filtering	40
3	Requirements on filters for multiresolution	44
3.1	Basic requirements for the scaling function	45
3.2	Wavelet definition	45
3.3	Orthonormality	47
3.4	Summary of conditions for multiresolution	48
3.5	Sufficiency of conditions	48
3.6	Construction of compactly supported orthonormal wavelets	50
3.7	Some shortcomings of compactly supported orthonormal bases	52
4	Approximation properties	52
4.1	Approximation from multiresolution spaces	52
4.2	Approximation using the largest wavelet coefficients	55
4.3	Local regularity	55
5	Extensions of orthonormal wavelet bases	56
5.1	Orthogonalization	56
5.2	Biorthogonal wavelets	57
5.3	Examples	59
5.4	Semiorthogonal wavelets	60
5.5	Other extensions of wavelets	60
5.6	Wavelets on intervals	60

1	Wavelets and signal compression	63
1.1	The need for compression	63
1.2	General idea	64
1.3	Error measure	66
1.4	Theory of wavelet compression	66
1.5	Image compression	67
1.6	Video compression	72
2	Wavelets and image processing	73

2.1	General idea	73
2.2	Multiscale edge detection and reconstruction	73
2.3	Enhancement	76
2.4	Others	77

IV Curves and Surfaces – *Leena-Maija Reissell, Tony D. DeRose* 79

1	Wavelets in Curve Representation (L-M. Reissell)	79
1.1	Notation	80
1.2	Basic properties	81
1.3	Curve segment trees	83
1.4	Choosing a wavelet	85
2	Wavelets with Interpolating Scaling Functions (L-M. Reissell)	86
2.1	The construction	86
2.2	The pseudocoiflet family P_{2N}	88
2.3	Example	90
3	Multiresolution Analysis for Surfaces of Arbitrary Topological Type (T. DeRose)	93
3.1	Introduction	93
3.2	A preview of the method	94
3.3	Our view of multiresolution analysis	95
3.4	Nested linear spaces through subdivision	95
3.5	Inner products over subdivision surfaces	99
3.6	Multiresolution analysis based on subdivision	100
3.7	Examples	103
3.8	Summary	104

V Wavelet Radiosity – *Peter Schröder* 107

1	Introduction	107
1.1	A Note on Dimensionality	108
2	Galerkin Methods	108
2.1	The Radiosity Equation	108
2.2	Projections	110
3	Linear Operators in Wavelet Bases	111
3.1	Standard Basis	112
3.2	Vanishing Moments	114
3.3	Calderón-Zygmund Operators and Sparsity	115
3.4	Non-Standard Basis	115
4	Wavelet Radiosity	116
4.1	Galerkin Radiosity	116
4.2	Hierarchical Radiosity	118

4.3	Algorithms	120
4.4	$O(n)$ Sparsity	125
5	Conclusion	127

VI More Applications – *Michael F. Cohen, Wim Sweldens, Alain Fournier* 129

1	Hierarchical Spacetime Control of Linked Figures (M. F. Cohen)	129
1.1	Introduction	129
1.2	System overview	131
1.3	Wavelets	131
1.4	Implementation	134
1.5	Results	135
1.6	Conclusion	136
2	Wavelets and Integral and Differential Equations (W. Sweldens)	137
2.1	Integral equations	137
2.2	Differential equations	138
3	Light Flux Representations (A. Fournier)	139
4	Fractals and Wavelets (Alain Fournier)	141
4.1	Fractal Models	141
4.2	Fractional Brownian Motion	141
4.3	Stochastic Interpolation to Approximate fBm	142
4.4	Generalized Stochastic Subdivision	142
4.5	Wavelet Synthesis of fBm	143

VII Pointers and Conclusions 147

1	Sources for Wavelets	147
2	Code for Wavelets	147
3	Conclusions	149

Bibliography 151

Index 161

1 Prolegomenon

These are the notes for the Course #11, **Wavelets and their Applications in Computer Graphics** given at the Siggraph '94 Conference. The lecturers and authors of the notes are (in alphabetical order) Michael Cohen, Tony DeRose, Alain Fournier, Leena-Maija Reissell, Peter Schröder and Wim Sweldens.

Michael Cohen is an Assistant Professor at the Department of Computer Science at Princeton University. He is one of the originators of the radiosity approach for global illumination. He has used in his own research wavelet techniques for curve modelling and hierarchical space-time control.

Tony DeRose is Associate Professor at the Department of Computer Science at the University of Washington. His main research interests are computer aided design of curves and surfaces, and he has applied wavelet techniques in particular to multiresolution representation of surfaces.

Alain Fournier is a Professor in the Department of Computer Science at the University of British Columbia. His research interests include modelling of natural phenomena, filtering and illumination models. His interest in wavelets derived from their use to represent light flux and to compute local illumination within a global illumination algorithm he is currently developing.

Leena-Maija Reissell is a Research Associate in Computer Science at UBC, on leave from XOX Corporation, Minneapolis, Minnesota. Ms Reissell is currently conducting research in curve and surface approximation with wavelet bases.

Peter Schröder is concluding his doctoral studies in Computer Science at Princeton University. His research activities have included dynamic modelling for computer animation, massively parallel graphics algorithms, global illumination algorithms, and most recently the application of wavelets to hierarchical radiosity algorithms. His thesis research concerns the design of wavelet based algorithms for the general global illumination problem.

Wim Sweldens is a Research Assistant of the Belgian National Science Foundation at the Department of Computer Science of the Katholieke Universiteit Leuven, and a Research Fellow at the Department of Mathematics of the University of South Carolina. His research interests include the construction of non-algebraic wavelets and their applications in numerical analysis and image processing. He is one of the regular editors the Wavelet Digest.

In the past few years *wavelets* have been developed both as a new analytic tool in mathematics and as a powerful source of practical tools for many applications from differential equations to image processing.

Wavelets and wavelet transforms are important to researchers and practitioners in computer graphics because they are a natural step from classic Fourier techniques in image processing, filtering and reconstruction, but also because they hold promises in shape and light modelling as well. It is clear that wavelets and wavelet transforms can become as important and ubiquitous in computer graphics as spline-based technique are now.

This course is intended to give the necessary mathematical background on wavelets, and explore the main applications, both current and potential, to computer graphics. The emphasis is put on the connection between wavelets and the tools and concepts which should be familiar to any skilled computer graphics person: Fourier techniques, pyramidal schemes, spline representations. We also tried to give a representative sample of recent research results, most of them presented by their authors.

The main objective of the course (through the lectures and through these notes) is to provide enough background on wavelets so that a researcher or skilled practitioner in computer graphics can understand the nature and properties of wavelets, and assess their suitability to solve specific problems in computer graphics. Our goal is that after the course and/or the study of these notes one should be able to access the basic mathematical literature on wavelets, understand and review critically the current computer graphics literature using them, and have some intuition about the pluses and minuses of wavelets and wavelet transform for a specific application.

We have tried to make these notes quite uniform in presentation and level, and give them a common list of references, pagination and style. At the same time we hope you still hear distinct voices. We have not tried to eradicate redundancy, because we believe that it is part and parcel of human communication and learning. We tried to keep the notation consistent as well but we left variations representative of what is normally found in the literature. It should be noted that the references are by no mean exhaustive. The literature of wavelets is by now huge. The entries are almost exclusively references made in the text, but see Chapter VI for more pointers to the literature.

The CD-ROM version includes an animation (720 frames) made by compressing (see Chapter III) and reconstructing 6 different images (the portraits of the lecturers) with six different wavelet bases. The text includes at the beginning of the first 6 chapters four frames (at 256×256 resolution originally) of each sequence. This gives an idea (of course limited by the resolution and quality of the display you see them on) of the characteristics and artefacts associated with the various transforms. In order of appearance, the sequences are Alain Fournier with Adelson bases, Leena-Maija Reissell with pseudo-coiflets, Wim Sweldens with Daubechies 8, Tony DeRose with Battle-Lemarié, Peter Schröder with Haar and Michael Cohen with coiflets 4. All of these (with the exception of Adelson) are described in the text.

Besides the authors/lecturers, many people have helped put these notes together. Research collaborators are identified in the relevant sections, some of them as co-authors. The latex/postscript version of these notes have been produced at the Department of Computer Science at the University of British Columbia, and Chris Romanzin has been instrumental in bringing them into existence. Without him they would be a disparate collection of individual sections, and Alain Fournier's notes would be in troff. Bob Lewis, also at UBC, has contributed greatly to the content of the first section, mostly through code and general understanding of the issues. The images heading the chapters, and the animation found on the CD-ROM were all computed with his code (also to be found on the disc -see Chapter VII). Parag Jain implemented an interactive program which was useful to explore various wavelet image compressions. Finally we want to thank Stephan R. Keith, the production editor of the CD-ROM, who was most helpful, patient and efficient as he had to deal with dozens of helpless, impatient and scattered note writers.

Alain Fournier

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