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Course Notes

11

WAVELETS AND THEIR APPLICATIONS IN COMPUTER GRAPHICS

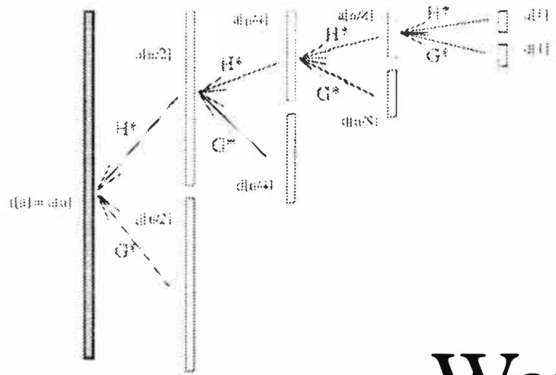
Organizer
Alain Fournier
University of British Columbia

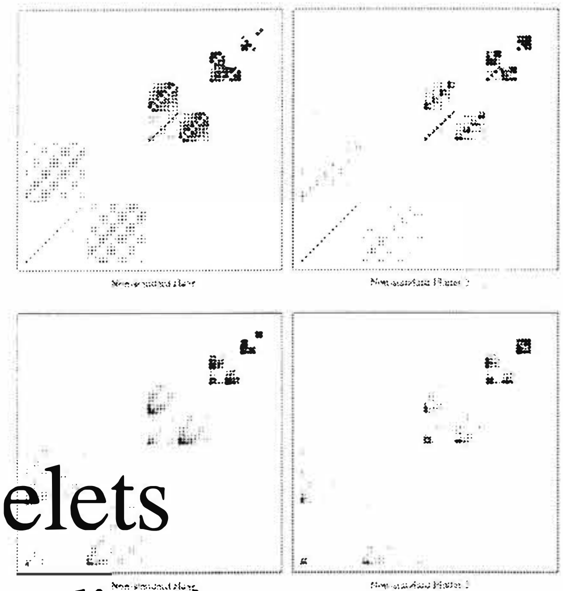
Lecturers
Leena-Maija Reissell
University of British Columbia

Peter Schroder
Princeton University

Michael F. Cohen
Princeton University

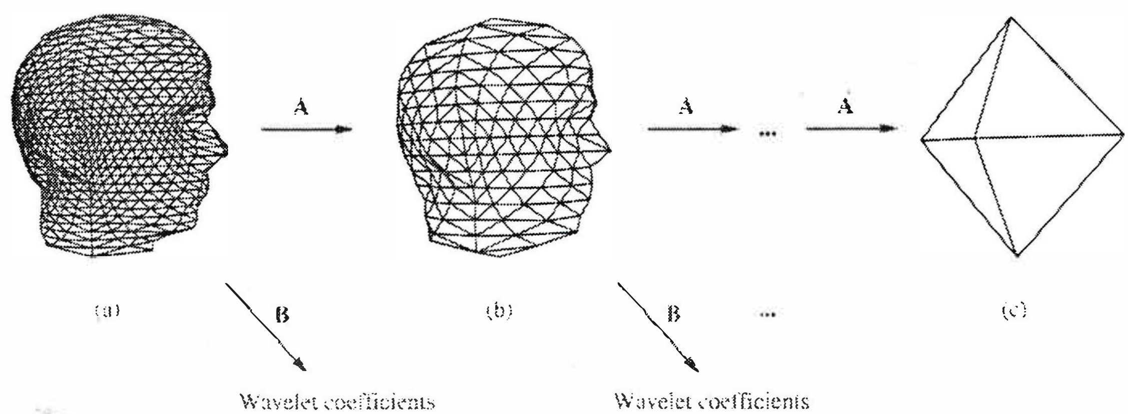
SIGGRAPH '94 Course Notes



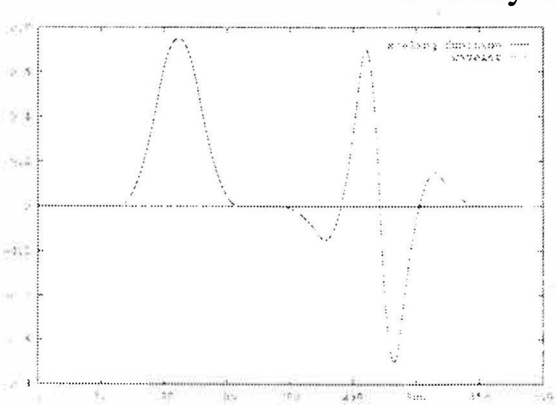


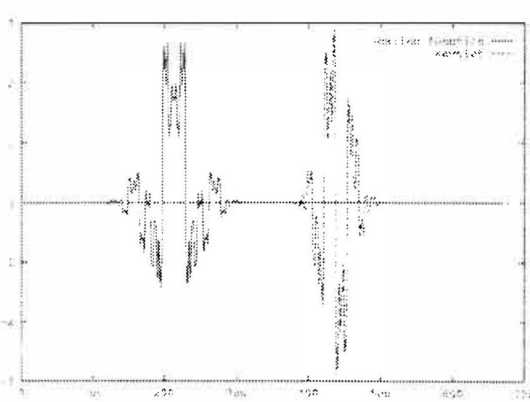
Wavelets

and their Applications in Computer Graphics



Organizer:
Alain Fournier
University of British Columbia





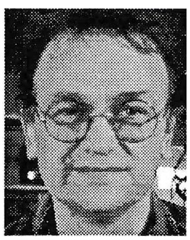
Nothing to do with sea or anything else.
Over and over it vanishes with the wave.

– Shinkichi Takahashi

Lecturers

Michael F. Cohen
Department of Computer Scie
Princeton Universit
35 Olden St.
Princeton, NJ 08544-20
mfc@cs.princeton.edu

Tony D. DeRose
Department of Computer Science and Engineering FR-35
University of Washington
Seattle, Washington 98195
derose@cs.washington.edu



Alain Fournier
Department of Computer Science
University of British Columbia
2366 Main Mall
Vancouver, British Columbia V6T 1Z4
fournier@cs.ubc.ca

Leena-Maija Reissell
Department of Computer Science
University of British Columbia
2366 Main Mall
Vancouver, British Columbia V6T 1Z4
reissell@cs.ubc.ca

Peter Schröder
Department of Computer Science
Princeton University
35 Olden St.
Princeton, NJ 08544-2087
ps@cs.princeton.edu

Wim Sweldens
Department of Mathematics
University of South Carolina
Columbia, SC 29208
sweldens@math.scarolina.edu

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N : Preamble

1 Prolegomenon

These are the notes for the Course #11, **Wavelets and their Applications in Computer Graphics** given at the Siggraph '94 Conference. The lecturers and authors of the notes are (in alphabetical order) Michael Cohen, Tony DeRose, Alain Fournier, Leena-Maija Reissell, Peter Schröder and Wim Sweldens.

Michael Cohen is an Assistant Professor at the Department of Computer Science at Princeton University. He is one of the originators of the radiosity approach for global illumination. He has used in his own research wavelet techniques for curve modelling and hierarchical space-time control.

Tony DeRose is Associate Professor at the Department of Computer Science at the University of Washington. His main research interests are computer aided design of curves and surfaces, and he has applied wavelet techniques in particular to multiresolution representation of surfaces.

Alain Fournier is a Professor in the Department of Computer Science at the University of British Columbia. His research interests include modelling of natural phenomena, filtering and illumination models. His interest in wavelets derived from their use to represent light flux and to compute local illumination within a global illumination algorithm he is currently developing.

Leena-Maija Reissell is a Research Associate in Computer Science at UBC, on leave from XOX Corporation, Minneapolis, Minnesota. Ms Reissell is currently conducting research in curve and surface approximation with wavelet bases.

Peter Schröder is concluding his doctoral studies in Computer Science at Princeton University. His research activities have included dynamic modelling for computer animation, massively parallel graphics algorithms, global illumination algorithms, and most recently the application of wavelets to hierarchical radiosity algorithms. His thesis research concerns the design of wavelet based algorithms for the general global illumination problem.

Wim Sweldens is a Research Assistant of the Belgian National Science Foundation at the Department of Computer Science of the Katholieke Universiteit Leuven, and a Research Fellow at the Department of Mathematics of the University of South Carolina. His research interests include the construction of non-algebraic wavelets and their applications in numerical analysis and image processing. He is one of the regular editors the Wavelet Digest.

In the past few years *wavelets* have been developed both as a new analytic tool in mathematics and as a powerful source of practical tools for many applications from differential equations to image processing.

Wavelets and wavelet transforms are important to researchers and practitioners in computer graphics because they are a natural step from classic Fourier techniques in image processing, filtering and reconstruction, but also because they hold promises in shape and light modelling as well. It is clear that wavelets and wavelet transforms can become as important and ubiquitous in computer graphics as spline-based technique are now.

This course is intented to give the necessary mathematical background on wavelets, and explore the main applications, both current and potential, to computer graphics. The emphasis is put on the connection between wavelets and the tools and concepts which should be familiar to any skilled computer graphics person: Fourier techniques, pyramidal schemes, spline representations. We also tried to give a representative sample of recent research results, most of them presented by their authors.

The main objective of the course (through the lectures and through these notes) is to provide enough background on wavelets so that a researcher or skilled practitioner in computer graphics can understand the nature and properties of wavelets, and assess their suitability to solve specific problems in computer graphics. Our goal is that after the course and/or the study of these notes one should be able to access the basic mathematical literature on wavelets, understand and review critically the current computer graphics literature using them, and have some intuition about the pluses and minuses of wavelets and wavelet transform for a specific application.

We have tried to make these notes quite uniform in presentation and level, and give them a common list of references, pagination and style. At the same time we hope you still hear distinct voices. We have not tried to eradicate redundancy, because we believe that it is part and parcel of human communication and learning. We tried to keep the notation consistent as well but we left variations representative of what is normally found in the literature. It should be noted that the references are by no mean exhaustive. The literature of wavelets is by now huge. The entries are almost exclusively references made in the text, but see Chapter VI for more pointers to the literature.

The CD-ROM version includes an animation (720 frames) made by compressing (see Chapter III) and reconstructing 6 different images (the portraits of the lecturers) with six different wavelet bases. The text includes at the beginning of the first 6 chapters four frames (at 256×256 resolution originally) of each sequence. This gives an idea (of course limited by the resolution and quality of the display you see them on) of the characteristics and artefacts associated with the various transforms. In order of appearance, the sequences are Alain Fournier with Adelson bases, Leena-Maija Reissell with pseudo-coiflets, Wim Sweldens with Daubechies 8, Tony DeRose with Battle-Lemarié, Peter Schröder with Haar and Michael Cohen with coiflets 4. All of these (with the exception of Adelson) are described in the text.

Besides the authors/lecturers, many people have helped put these notes together. Research collaborators are identified in the relevant sections, some of them as co-authors. The latex/postscript version of these notes have been produced at the Department of Computer Science at the University of British Columbia, and Chris Romanzin has been instrumental in bringing them into existence. Without him they would be a disparate collection of individual sections, and Alain Fournier's notes would be in troff. Bob Lewis, also at UBC, has contributed greatly to the content of the first section, mostly through code and general understanding of the issues. The images heading the chapters, and the animation found on the CD-ROM were all computed with his code (also to be found on the disc -see Chapter VII). Parag Jain implemented an interactive program which was useful to explore various wavelet image compressions. Finally we want to thank Stephan R. Keith, the production editor of the CD-ROM, who was most helpful, patient and efficient as he had to deal with dozens of helpless, impatient and scattered note writers.

Alain Fournier

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