

MATH FOR SIGGRAPH

COURSE # 23

CO-CHAIRS:

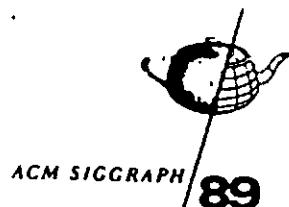
Ken Shoemake
Xerox PARC

Tony DeRose
University of Washington

SPEAKERS:

James Kajiya
California Institute of Technology

John Platt
California Institute of Technology



Boston, Massachusetts
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Math for SIGGRAPH

Introduction

I hope you will enjoy this new course, and profit from these notes after the lectures are over. Each of the speakers has worked for some time with the topics they are presenting. The level of presentation varies from elaborate to brisk, but always aims to convey the essentials, the core concepts and intuitions behind the math.

Some of the papers in these notes have not been published elsewhere, and some are hard to obtain. This is a chronic problem, caused in part by the restrictions of the SIGGRAPH Proceedings. So enjoy.

The first topic is digital transforms and filters. The presentation is an attempt at merging my experience in the digital audio realm with the needs of computer graphics. Every raster image incorporates this theory, which I consider core material. Unfortunately, the usual presentations are designed for either engineering or image processing students, not computer graphicists. This is an alternative to those expositions, which, hopefully, is more accessible.

The second topic is affine and Euclidean geometry, more core material. Tony DeRose has successfully taught students at the University of Washington using this alternative to the more common, but also more confusing, "everything is a homogeneous matrix" approach. This is followed by a presentation of some recent results by Tony and Maureen Stone, using affine transformations to help classify cubic curves.

Next, Jim Kajiya presents an introduction to differential geometry, based on a course he has taught at Caltech for the last two years. Jim is noted for his facility with less familiar math, and differential geometry certainly qualifies in that category for most computer graphicists. While this material obviously applies to the curves and surfaces used for modeling, it is also of use for differential equations of motion. This is perhaps the most difficult material in the course, but a worthwhile investment.

From that foundation, I will follow with a discussion of quaternions, a mathematical tool I introduced to SIGGRAPH in 1985. Quaternions are used as a parametrization of rotations. They have a rich structure, which requires differential geometry to fully appreciate. Researchers continue to find that quaternions make life easier when dealing with rotation.

The concluding speaker is John Platt, who has recently acquired his Ph.D. from Caltech. He will discuss the numerical methods needed to implement various forms of physically-based modelling and animation – his thesis topic. A conversation with John makes even the more difficult algorithms seem straightforward. Numerical analysis is another one of those essential ingredients of graphics algorithms which is too frequently neglected, with unfortunate results. To inspire you to master this material, John will end the day with a presentation of his research in using rate-controlled constraints. You can enjoy his pictures, but you won't be able to make your own without learning numerical methods.

– Ken Shoemake

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Schedule

8:30	Shoemake	Introduction and overview
8:45	Shoemake	Digital transforms and filters
10:15	—	Break
10:30	DeRose	Coordinate-free geometry
11:40	DeRose	Characterizing cubic curves
12:00	—	Lunch
1:30	Kajiya	Differential geometry
2:50	Shoemake	Quaternions
3:30	—	Break
3:45	Platt	Numerical methods
4:40	Platt	Rate-controlled constraints
5:00	—	END

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