Fractals and an Art for the Sake of Science

Benoit B. Mandelbrot

ABSTRACT

A new form of art redefines the boundary between 'invention' and 'discovery', as understood in the sciences, and 'creativity', as understood in the plastic arts. Can pure geometry be perceived by the 'man in the street' as beautiful? To be more specific, can a shape that is defined by a simple equation or a simple rule of construction be perceived by people other than geometers as having aesthetic value—namely, as being at least surprisingly decorative—or perhaps even as being a work of art? When the geometric shape is a fractal, the answer is yes. Even when fractals are taken 'raw', they are attractive. They lend themselves to 'painting by numbers' that is surprisingly effective, even in the hands of the rank amateur. And the true artist's sensibility finds them a novel and attractive support.

The artist and the artisan are often hard to tell apart. For example, objects that were in principle meant to be utilitarian—be it folk architecture, religious imagery, or drawings and photographs of flowers, birds or water eddies—often end up being regarded as genuine works of art. It may become hard to distinguish them from works in which science was used almost as an excuse for artistic creativity. Thus, art faces us with broad possibilities. We are presented with innumerable works of art for the sake of commerce: objects have been commissioned under precise specifications to be useful—to decorate, to educate, to flatter, to entertain, to impress or to persuade. We are also presented with a few works created strictly as art for art's sake. And we also know of many possibilities that lie, so to say, in-between.

Does mathematics relate in any way to these familiar forms of plastic art? The classic shapes of geometry are hailed for their conceptual beauty, but they seem mostly to reside in the imagination of skilled practitioners. Although the popular poet Edna Saint Vincent Millay proclaimed that "Euclid Gazed on Beauty Bare" and although Euclid's geometry was of central importance to painters of the Italian Renaissance during the brief period when perspective was being 'invented', to the eye of those unschooled in mathematics, the beauty of Euclid's geometry is bare and dry to a fault. At the least it lacks scope and visual variety when compared with those excesses of either nature or the fine arts, which everyone seems tempted to call 'baroque' or 'organic'.

Today, however, there is more to geometry than Euclid. During the 1970s it was my privilege to conceive and develop fractal geometry [1], a body of thoughts, formulas and pictures that may be called either a new *geometry of nature* or a new *geometric language*. And the reason why it is worth discussing here is that I have discovered that, most surprisingly and without any prodding, this new geometric language has given rise to a new form of art. I propose here to make a few disjointed comments on its account. Many readers are bound to be familiar with fractal art, and the volume in which this paper appears may well contain some new examples from the 1989 SIGGRAPH show; nevertheless, close familiarity with the subject is not expected from the reader.

The bulk of fractal art has not been commissioned for any commercial purpose, even though all the early work was done at IBM. And it has not necessarily been touched by esthetic sensibility. Therefore, we shall argue that fractal geometry appears to have created a new category of art, next to art for art's sake and art for the sake of commerce: art for the sake of science (and of mathematics).

Fractal art for the sake of science is indissolubly based on the use of computers. It could not possibly have arisen before the hardware was ready and the software was being developed; that is, before the decade of the seventies. What a profound irony that this new geometry, which everyone seems spontaneously to describe as 'baroque' and 'organic', should owe its birth to an unexpected but profound new match between those two symbols of the inhuman, the dry, and the technical: namely, between mathematics and the computer.

Before we describe the peculiarities of fractal geometry in more detail, it is good, for the sake of contrast, to comment on examples of similar matches that have arisen in areas such as the study of water eddies and wakes. In these cases, the input in terms of reasoning and programs is extremely complicated, perhaps more complicated even than the output. In fact, one may argue that, overall, complication does not increase but changes over from being purely conceptual to being partly visual, a change that is important practically and interesting conceptually. Fractal geometry, however, gives us something quite different. In fractal geometry, the inputs are typically so extraordinarily simple as to look positively simple-minded. The outputs, to the contrary, can be spectacularly complex. Again, while a contribution from an artistic sensibility is not necessary, it is well rewarded.

Let us hasten to raise a question. Since the inputs are so simple, why is it that fractal art failed to appear earlier and in more traditional ways? The answer lies in a 'Catch 22' situation. To draw the simplest fractal picture 'by hand' would have been feasible in principle, but would have required many person-years and would have been ridiculously expensive. Consequently, no one would have considered undertaking this task without having a fair advance knowledge of the result; yet the result could not even be suspected until one actually had performed the task. And a sure way of being discouraged from ever undertaking it would have been to begin with any one of the various definitions of fractals. Here is one informal definition I often use:

Fractals are geometric shapes that are equally complex in their details as in their overall form. That is, if a piece of a fractal is suitably magnified to become of the same size as the whole, it should look

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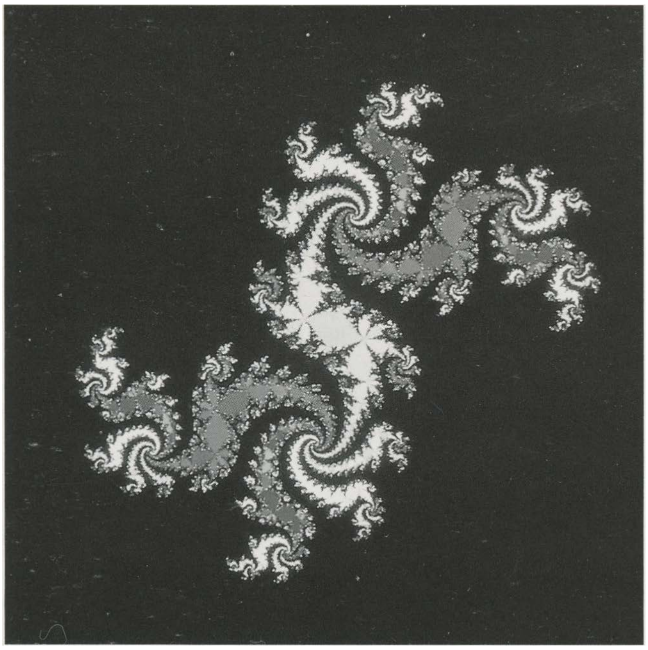


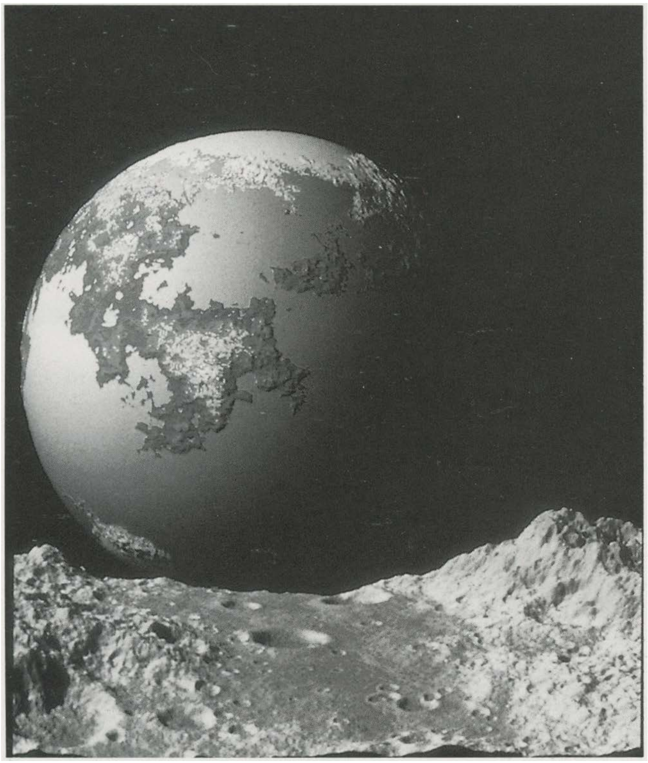


Fig. 1. The two faces of fractal art. (above) M. R. Laff and A. V. Norton, *Fractal Dragon*, 1982. (below) R. F. Von, *Fractal Planetrise*, 1982. This *Fractal Dragon* and *Fractal Planetrise* may be the best known of all fractals, since they appear on the two halves of the jacket of *The Fractal Geometry of Nature* (Ref. [4]). Their being set as neighbors is meant to illustrate the basic fact that fractal art straddles the boundary between art that is, and is not, representational.

like the whole, either exactly, or perhaps only after a slight limited deformation.

Are we not right in the middle of dry geometric principles? An artist could expect nothing from fractals defined in this fashion, hence no one attempted to draw them carefully. The few old fractals that had been known under various names (and depicted for at least a century) are also the least interesting esthetically because one glance shows that everything about them has been obviously put in by hand; they are orderly to excess. These images, however, began to grow in number and in variety after they were picked up and made into the first few 'words' in the new geometric language of fractals. This happened with my first book in 1975 [2].

What were the needs that led me to single out a few of these monsters, calling them *fractals*, to add some of their close or distant kin, and then to build a geometric language around them? The original need happens to have been purely utilitarian. That links exist between usefulness and beauty is, of course, well known. What we call the beauty of a flower attracts the insects that will gather and spread its pollen. Thus the beauty of a flower is useful—even indispensable—to the survival of its species. Similarly, it was the attractiveness of the fractal images that first brought them to the attention of many colleagues and then of a wide world.

Let me tell how this started happening. In the 1960s, the basic idea of the theory of fractals was already present in my mind, having been devised to study such phenomena as the erratic behavior of stock prices, turbulence in fluids, the persistence of the discharges of the Nile, and the clustering of galaxies, which manifests itself with the presence of great intergalactic empty spaces. But society seemed to think that my theories, their mathematical techniques and their goals were strange, as opposed to simply new. As a result, my attempts to make my thoughts accepted as sound seemed always to encounter a wall of hostility that words and formulas failed to circumvent.

One day it became necessary to convince Walter Langbein, the editor of a water resources journal, to accept a paper I had co-authored. He was a skilled and able scientist, but not one to gamble on wild, unproven ideas. I decided to resort to a tactical detour, presenting him with two images in the hope that Langbein would find it impossible to distinguish between reality and 'forgeries' that were based solely upon an early fractal theory. If this were to happen, he would no longer be able to view this theory as irrelevant to his work, he could not and would not reject our paper outright, and he might eventually accept fractals. This is indeed what happened: the detour through the eye turned out to be successful, and its offspring grew beyond expectation.

What happened next to fractal art as it evolved brings us to the traditional dichotomy between representational and nonrepresentational art. In the well-recognized forms of art, this dichotomy no longer seems so strongly etched, and fractal art straddles it very comfortably. The earliest explicit uses of fractals gave me the privilege of being the first person to tackle in a new way some problems that must be among the oldest that humanity had asked itself: how to obtain 'figures' that represent the shapes of mountains, clouds and rivers? It turns out that, when the representation of nature by fractal is perceived as successful, it also tends to be perceived as beautiful. Unquestionably, the fractal 'forgeries' of mountains and clouds are examples of representational art.

The skeptic will immediately raise another question. Is it not true that the colors used to render these mountains and clouds are chosen by rules that have nothing to do with any geometry? If this is so, these 'forgeries' are not purely fractal. What precise role, then, does color play in the acceptance of what you call 'fractal art'? This may sound like a very strong objection, but in fact it is easy to answer.

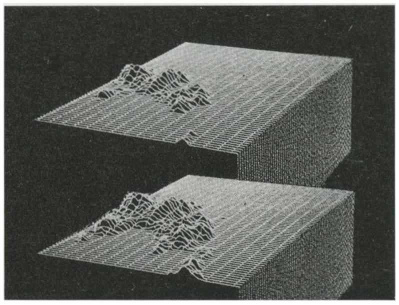
First of all, the question did not and could not arise with the first fractal pictures, simply because they were in black and white. I might also add that in many cases this supposedly obsolete palette is the one I continue to favor.

When the use of color did arise, Richard Voss and I worried that it might detract from our primary concern with the geometry. Thus, initially he decided to color his art simply, as in the *London Times World Atlas*; but in landscapes viewed from an angle instead of the zenith, this proved to be visually unacceptable. However, we continued to avoid excessive artistic intervention, and Voss kept his esthetic urges under the tightest of control. This, in my opinion, helped fractal geometry make its intended point. Once that point was achieved, however, a completely different situation was created in which reserve was no longer an overriding obligation. In the recent crop of pictures by F. Kenton Musgrave, 'SIGGRAPH tricks' are allowed, but one *absolute* constraint remains. Every surface that is depicted must be a fractal surface, and all commands that are used to improve the rendering must be global commands. To 'fix' an unsatisfactory corner of a piece by a local patch is not permitted. Many computer artists would find this constraint to be quixotic, but it is essential if fractal art is to preserve its integrity.

While dealing with fractals intended as forgeries of nature, we found that cases soon began to multiply in which this intent failed. The result, however, remained just as beautiful, and occasionally even more so. Happy errors! Furthermore, a person fascinated by shapes could not avoid forgetting on occasion the original goals of the fractal geometry *of nature* and would play on with fractal algorithms just to find where they might lead. Thus as a fractal model of mountains is deformed by changing the values given to one or a few numbers that characterize the fractal's form, the picture becomes less and less 'realistic' as a mountain and gradually becomes altogether 'surreal'.

Even more striking surrealism prevails within the second major aspect of fractal geometry. Fractal 'dragons', of which the 'oldest' is reproduced here (see Fig. 1)—and of which millions seem to have been drawn since—have never been meant to represent anything in nature. Their intended usefulness concerned mathematics, since they helped me investigate a process called the 'dynamics of iteration'. Early in the century, the mathematicians Pierre Fatou and Gaston Julia had found that this process presents a deep and surprisingly intellectual challenge. Then for 60 years hardly anyone touched the problem because even the most brilliant mathematicians, when working alone with the proverbial combination of pencil-and-paper and mental images, found that its study had become too complicated to be managed. My fresh attack on iteration could rely upon the help of the computer, and it was effective: the new mathematical order was spectacular. For the purposes of this discussion, this does not matter at all, of course; but a side result does matter a great deal: the resulting balanced co-existence of order and chaos was found almost invariably to be beautiful.

As in the case of the fractal mountains, the new iteration-generated fractals were already perceived to be



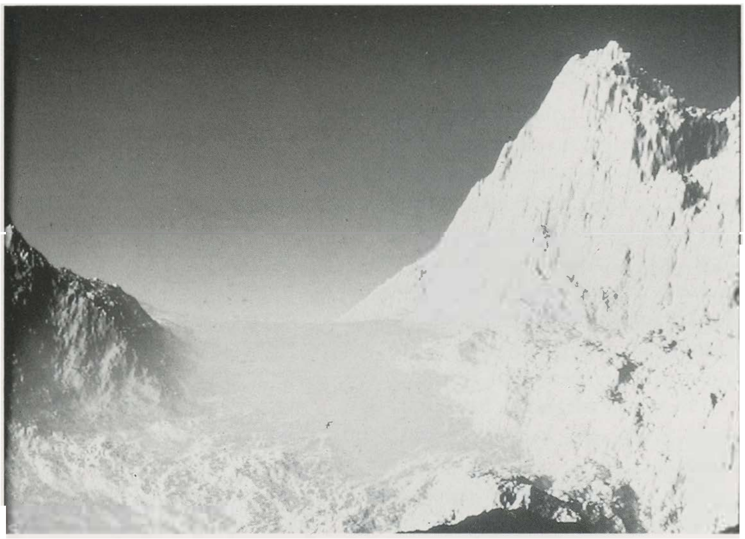


Fig. 2. Fractal landscapes. These illustrations exemplify three of many successive stages in the development of fractal landscapes. One may call these stages, respectively, 'archaic', 'classic' and 'romantic'.

The archaic wire model illustration (above) was done by S. W. Handelman (1974), who was then my programmer at IBM at a time when our work was dominated by the extreme crudeness of the tools.

The classic illustration (below) is by Richard F. Voss of IBM (1985). It is an improved form of one in a series he prepared for my book of 1982 [4]. By then, the computer tools had become less obtrusive, and allowing fancy to take over was a genuine temptation. But fantasy had to be resisted because these pictures were primarily tools of scientific discourse. The wonder is that these extreme constraints should have allowed the emergence of Voss's masterpieces of subdued elegance.

The romantic illustration (see back cover) is by my present Yale student, F. Kenton Musgrave, and myself (1989). Today, wire models that are better than the archaic one take 1 second to be computed and drawn on a workstation, and the number of available colors has changed from being unmanageably small to being unmanageably large. The most innovative use of fractals now is to serve as support for an artist's inspiration and skill.

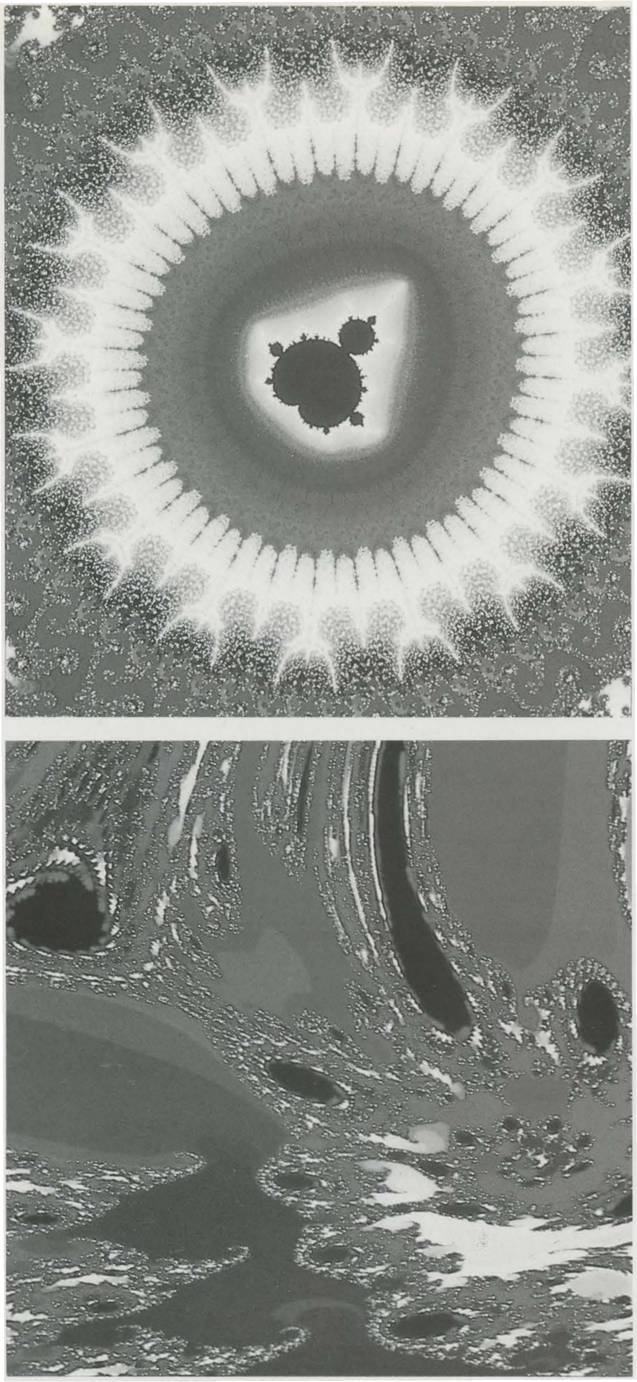


Fig. 3. Two fragments of the Mandelbrot set. The Mandelbrot set is explained in Refs [4], [7] and [8]. The first fragment (above; R. F. Voss, 1988) was selected so as to include, near its center, a small replica of the whole, with its obvious symmetries and repetitions, and even to include additional symmetries that are not present in the whole set. This fragment, therefore, leans too far towards orderliness. The second fragment (below; B. B. Mandelbrot), which is from a generalized, not the 'ordinary', Mandelbrot set, was selected to provide contrast since it is devoid of obvious symmetries.

beautiful in their original black and white. More precisely, the output of my work was a collection of numbers that in the early stages had to be reduced to two possibilities, to be represented by black and white. After color became involved, these numbers were first represented by colors chosen more or less at random by color-blind hackers. (An awful case of painting by numbers!) Yet even these fractals were, in a way, beautiful. But when the coloring was placed in the hands of a true artist, we began to see true wonders.

Our skeptical critic will come back at this point to remind us that fractals should share the credit for this art with both the computer and the programmerartist who frames the object and selects the colors. These last two factors are the ones usually considered central to computer art; hence the critic's point concerns the significance of the fractal's additional input. In some cases (as in one of the illustrations of this paper) fractals' most obvious contribution is an obtrusive symmetry that may in fact be found to be very objectionable. In other cases, however, when the symmetry is hidden we see an interplay between strong order and just enough change and surprise. My readings on the meaning of art suggest that such an interplay is one of the basic prerequisites of plastic beauty.

To summarize, the altogether new feature brought in by fractal art is that the proper interplay between order and surprise need not be the result either of the imitation of nature or of human creativity, and it can result from something entirely different. The source of fractal art resides in the recognition that very simple mathematical formulas that seem completely barren may in fact be pregnant, so to speak, with an enormous amount of graphic structure. The artist's taste can only affect the selection of formulas to be rendered, the cropping and the rendering. Thus, fractal art seems to fall outside the usual categories of 'invention', 'discovery' and 'creativity'.

All this seems to have happened long ago, and today fractal geometry is so well established that young people are astonished to find that the 'father of fractal geometry' (as I am delighted to be called) is still alive. But I hope to live long enough to really understand what has happened.

References and Notes

1. The first three books on fractals are Refs [2], [3] and [4]. Among later books, I recommend Refs [7] and [8], which are of higher graphic quality than mine, though of narrower focus. Of the approximately 40 books that have by now been written on fractals, nearly all are for mathematicians and/or physicists, and Refs [4], [7] and [8] are the only books written in English for a broad audience. Ref. [5] belongs here because of the journal in which it appeared and because it is my earliest statement on the issue of fractals and esthetics, and Ref. [6] can be viewed as a commentary on the present paper.

A quandary and an Apology. It would have been nice also to recommend works with which I am less closely associated, but this would be very hard. Each quote creates one happy person and many unhappy ones. The time when my close associates were the only people involved with fractals passed years ago, and to write a balanced survey is not a task I enjoy.

2. B. B. Mandelbrot, *Les objets fractals* (Paris: Flammarion, 1975, 1984, 1989).

3. B. B. Mandelbrot, *Fractals: Form, Chance and Dimension* (San Francisco: W. H. Freeman and Company, 1977).

4. B. B. Mandelbrot, *The Fractal Geometry of Nature* (New York: W. H. Freeman and Company, 1982).

5. B. B. Mandelbrot, "Scalebound or Scaling Shapes: A Useful Distinction in the Visual Arts and the Natural Sciences", *Leonardo* **14**, No. 1, 45–47 (1981).

6. F. K. Musgrave and B. B. Mandelbrot, "Natura ex Machina," *IEEE Computer Graphics and Applications* **9**, No. 1, cover and 4–7 (1989).

7. H.-O. Peitgen and P. H. Richter, *The Beauty of Fractals* (New York: Springer, 1986).

8. H.-O. Peitgen and D. Saupe, eds., *The Science of Fractal Images* (New York: Springer, 1988).